# 2.2 Graphs of Equations in Two Variables

# Equation in Two Variables

- An equation in two variables, such as $y = x^2 + 1$, expresses a relationship between two quantities.

# Graph of an Equation in Two Variables

- A point $(x, y)$ satisfies the equation if it makes the equation true when the values for $x$ and $y$ are substituted into the equation.

  - For example, the point (3, 10) satisfies the equation $y = x^2 + 1$ because $10 = 3^2 + 1$.

  - However, the point (1, 3) does not, because $3 \neq 1^2 + 1$.

# The Graph of an Equation

- The graph of an equation in $x$ and $y$ is:
  - The set of all points $(x, y)$ in the coordinate plane that satisfy the equation.

# The Graph of an Equation

- The graph of an equation is a curve, just a general name, it could be a line, a box, but graphs in a 2D plane are called curves, in 3D, they are called surfaces.
- So, to graph an equation, we:

1. Plot as many points as we can.
2. Connect them by a smooth curve.

# Sketching a Graph by Plotting Points

- Sketch the graph of the equation
- $$2x - y = 3$$
  - We first solve the given equation for $y$ to get:

    $$y = 2x - 3$$

# Sketching a Graph by Plotting Points

- Use a table, put in the x values and the equation that will be y. Then the ordered pair

| $x$ | $y = 2x - 3$ | $(x, y)$ |
| --- | --- | --- |
| $-1$ | $-5$ | $(-1, -5)$ |
| $0$ | $-3$ | $(0, -3)$ |
| $1$ | $-1$ | $(1, -1)$ |
| $2$ | $1$ | $(2, 1)$ |
| $3$ | $3$ | $(3, 3)$ |
| $4$ | $5$ | $(4, 5)$ |

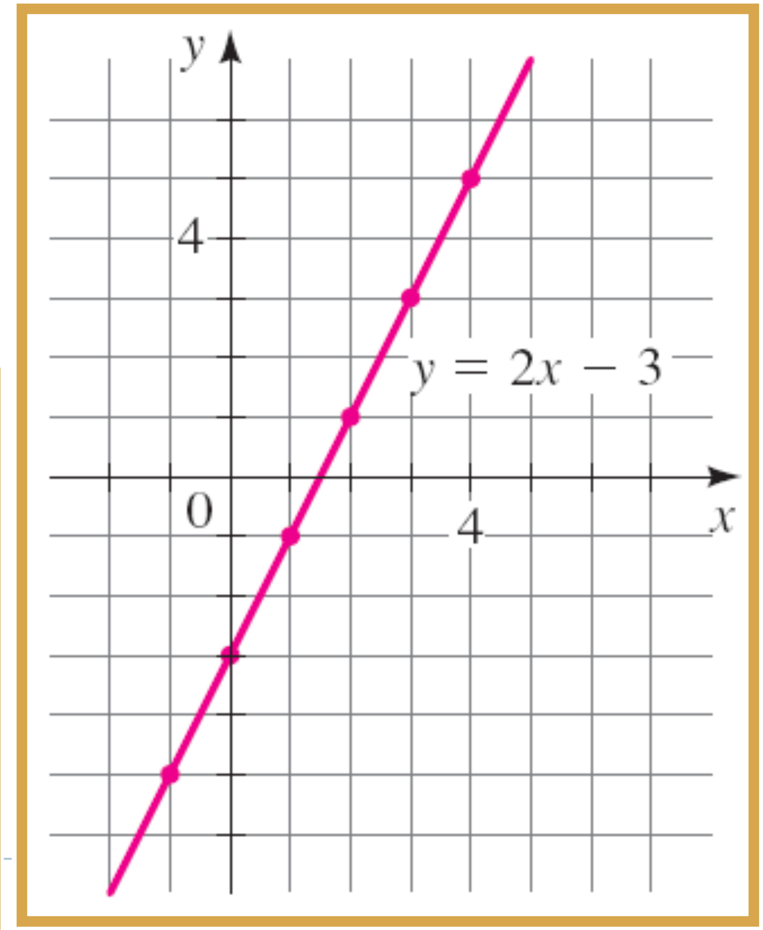
# Sketching a Graph by Plotting Points

- Of course, there are infinitely many points on the graph—and it is impossible to plot all of them.
  - But, the more points we plot, the better we can imagine what the graph represented by the equation looks like.

# Sketching a Graph by Plotting Points

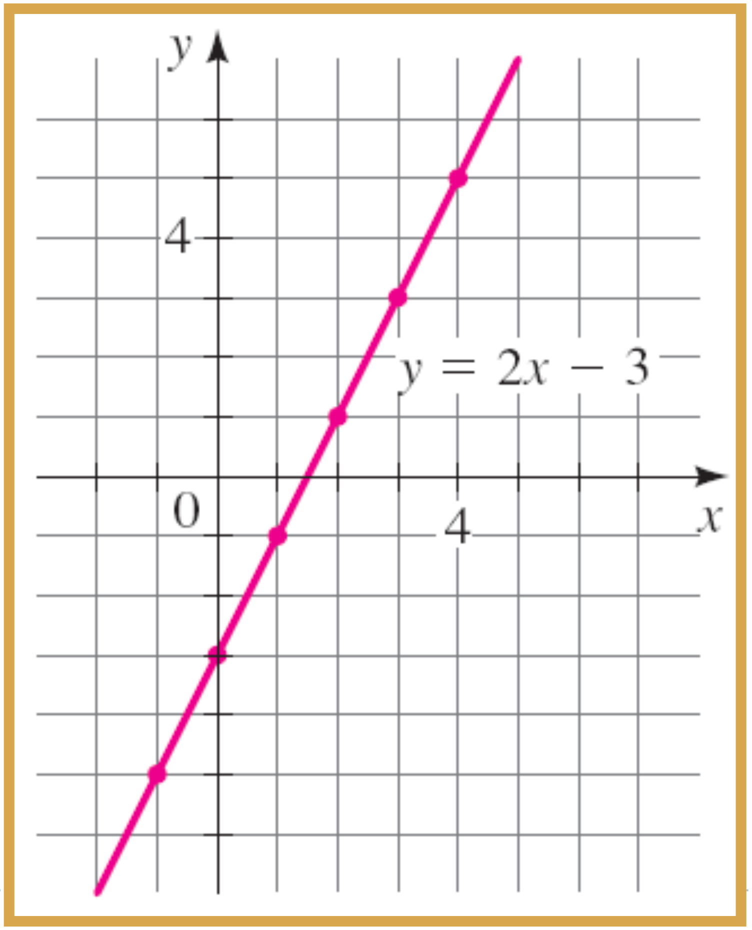
- Plot the points that work in the equation
  - As they appear to lie on a line, we complete the graph by joining the points by a line.

| $x$ | $y = 2x - 3$ | $(x, y)$ |
|---|---|---|
| $-1$ | $-5$ | $(-1, -5)$ |
| $0$ | $-3$ | $(0, -3)$ |
| $1$ | $-1$ | $(1, -1)$ |
| $2$ | $1$ | $(2, 1)$ |
| $3$ | $3$ | $(3, 3)$ |
| $4$ | $5$ | $(4, 5)$ |



# Sketching a Graph by Plotting Points

- The graph of this equation is indeed a line.



# Sketching a Graph by Plotting Points

- Sketch the graph of the equation

$$y = x^2 - 2$$

# Sketching a Graph by Plotting Points

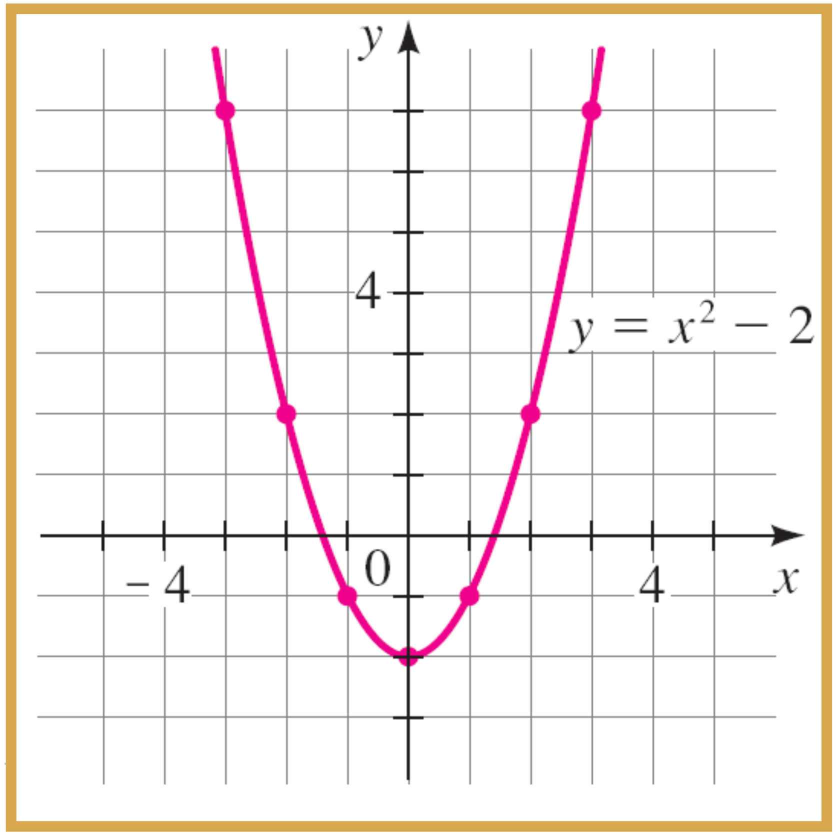
- Plug in some x and see what you get for y…In Ch 2.3, you will get your calculator to do this.

| $x$ | $y = x^2 - 2$ | $(x, y)$ |
|---|---|---|
| $-3$ | $7$ | $(-3, 7)$ |
| $-2$ | $2$ | $(-2, 2)$ |
| $-1$ | $-1$ | $(-1, -1)$ |
| $0$ | $-2$ | $(0, -2)$ |
| $1$ | $-1$ | $(1, -1)$ |
| $2$ | $2$ | $(2, 2)$ |
| $3$ | $7$ | $(3, 7)$ |

# Sketching a Graph by Plotting Points

- Plot the points and then connect them
  - A curve with this shape is called a parabola.

| $x$ | $y = x^2 - 2$ | $(x, y)$ |
|---|---|---|
| $-3$ | 7 | $(-3, 7)$ |
| $-2$ | 2 | $(-2, 2)$ |
| $-1$ | $-1$ | $(-1, -1)$ |
| 0 | $-2$ | $(0, -2)$ |
| 1 | $-1$ | $(1, -1)$ |
| 2 | 2 | $(2, 2)$ |
| 3 | 7 | $(3, 7)$ |



# E.g. 3—Graphing an Absolute Value Equation

- Sketch the graph of the equation

$$y = |x|$$

- Remember from 1.7
- y=-x
- y=x

# Graphing an Absolute Value Equation

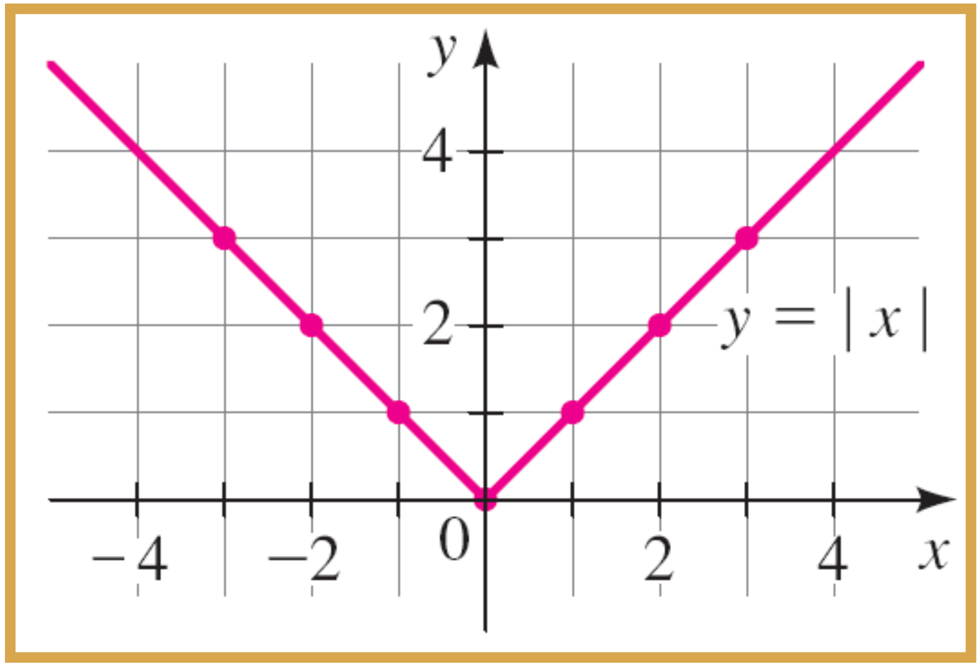
- Again, we make a table of values.

| $x$ | $y = \lvert x \rvert$ | $(x, y)$ |
|---|---|---|
| $-3$ | $3$ | $(-3, 3)$ |
| $-2$ | $2$ | $(-2, 2)$ |
| $-1$ | $1$ | $(-1, 1)$ |
| $0$ | $0$ | $(0, 0)$ |
| $1$ | $1$ | $(1, 1)$ |
| $2$ | $2$ | $(2, 2)$ |
| $3$ | $3$ | $(3, 3)$ |

# Graphing an Absolute Value Equation

- Tables are great and they make for flawless graphing….

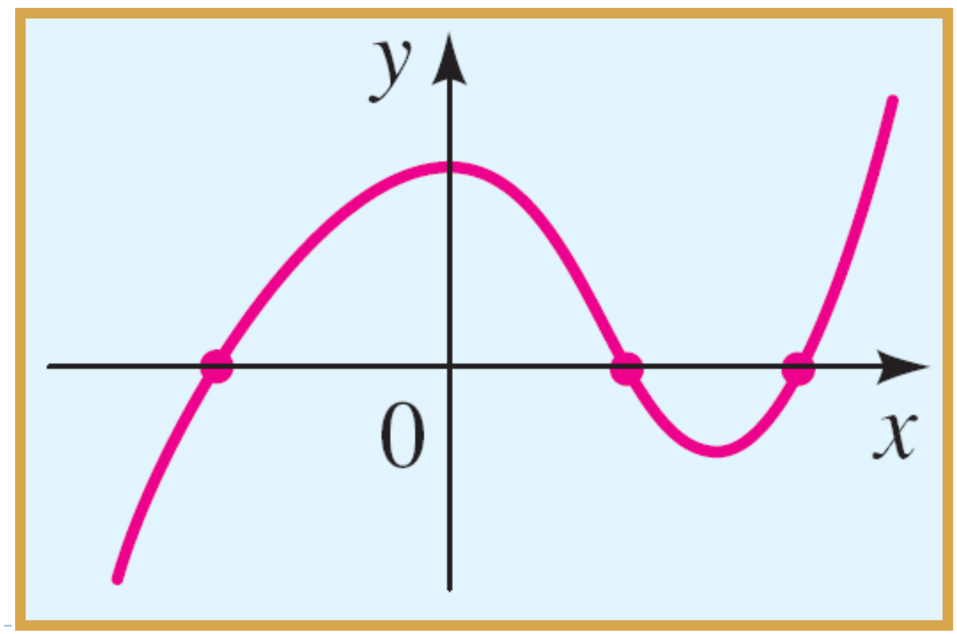
| $x$ | $y = \|x\|$ | $(x, y)$ |
|---|---|---|
| $-3$ | 3 | $(-3, 3)$ |
| $-2$ | 2 | $(-2, 2)$ |
| $-1$ | 1 | $(-1, 1)$ |
| 0 | 0 | $(0, 0)$ |
| 1 | 1 | $(1, 1)$ |
| 2 | 2 | $(2, 2)$ |
| 3 | 3 | $(3, 3)$ |



- Intercepts

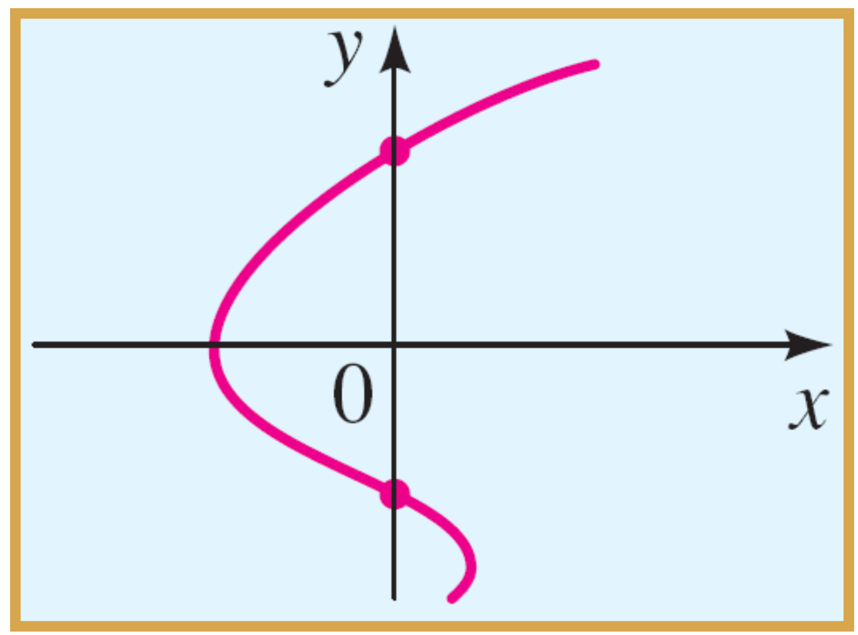
# x-intercepts

- Are the *x*-coordinates where a graph intersects the *x*-axis, they are called the *x*-intercepts of the graph.
  - They are obtained by setting $y = 0$ in the equation of the graph.



# Why? y-intercepts

- The *y*-coordinates of the points where a graph intersects the *y*-axis are called the *y*-intercepts of the graph.
  - They are obtained by setting $x = 0$ in the equation of the graph.



# Finding Intercepts

- Find the *x*- and *y*-intercepts of the graph of the equation

$$y = x^2 - 2$$

# Finding Intercepts

- To find the $x$-intercepts, we set $y = 0$ and solve for $x$.
  - Thus,

$$0 = x^2 - 2$$

$$x^2 = 2 \qquad \text{(Add 2 to each side)}$$

(Take the sq. root)

$$x = \pm\sqrt{2}$$

  - The $x$-intercepts are $\sqrt{2}$ and $-\sqrt{2}$

# Finding Intercepts

- To find the *y*-intercepts, we set $x = 0$ and solve for *y*.
  - Thus,

$$y = 0^2 - 2$$
$$y = -2$$

  - The *y*-intercept is –2.

- Circles

# Circles

- So far, we have discussed how to find the graph of an equation in $x$ and $y$.
- The converse problem is to find an equation of a graph—an equation that represents a given curve in the $xy$-plane.
- We can do this easily with circles because they have a very standard equation

# Circles

- As an example of this type of problem, let’s find the equation of a circle with radius $r$ and center $(h, k)$.

# Circles

- From the distance formula, we have:

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

$$(x-h)^2+(y-k)^2=r^2$$ (Square each side)

- This is the desired equation.

# Equation of a Circle—Standard Form

- An equation of the circle with center $(h, k)$ and radius $r$ is:

$$(x - h)^2 + (y - k)^2 = r^2$$

  - This is called the standard form for the equation of the circle.

# Equation of a Circle

- If the center of the circle is the origin (0, 0), then the equation is:

$$x^2 + y^2 = r^2$$

- h=0
- k=0
- r=> the radius

# Graphing a Circle

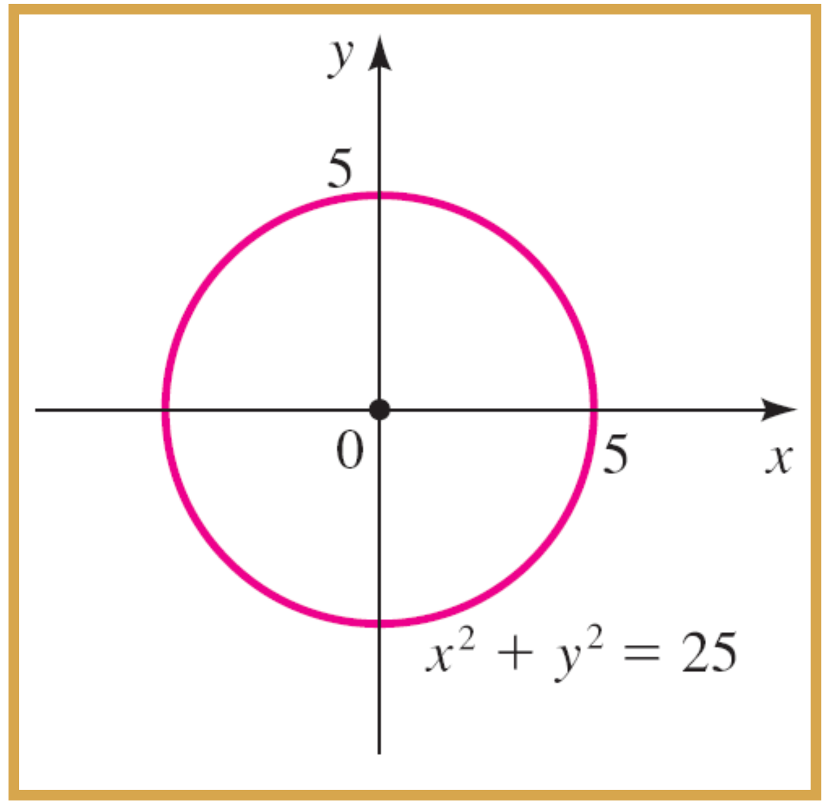
- Graph each equation.
- (You know these are circles, because of the standard form of a circle equation)

(a) $x^2 + y^2 = 25$

(b) $(x - 2)^2 + (y + 1)^2 = 25$

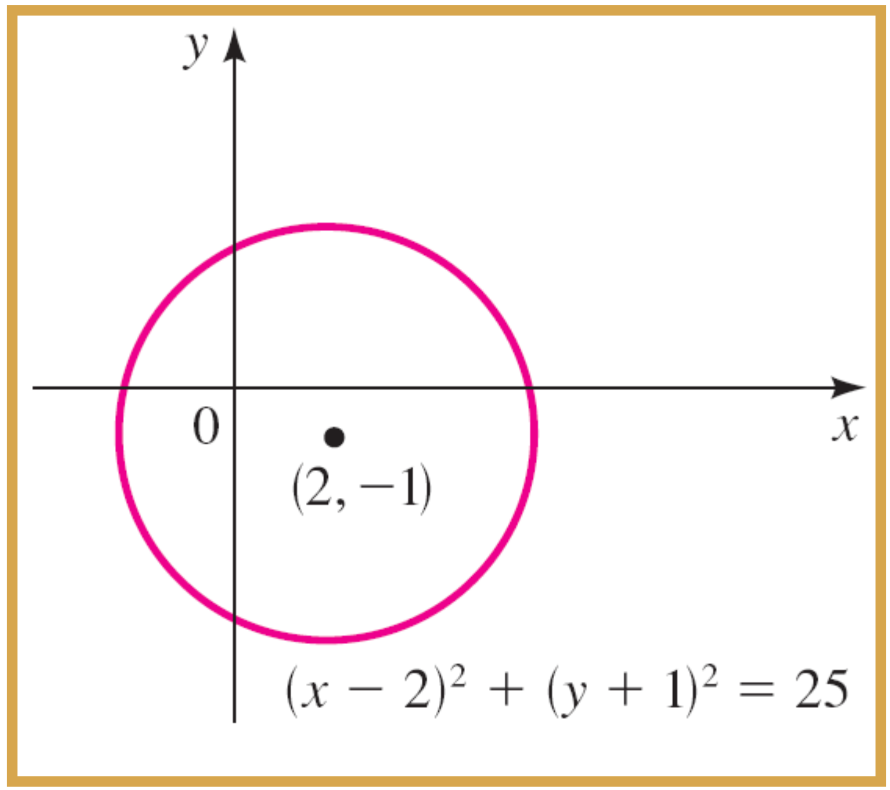
# Graphing a Circle

- Rewriting the equation as $x^2 + y^2 = 5^2$, we see that that this is an equation of:
  - The circle of radius 5 centered at the origin.



# Graphing a Circle

- Rewriting the equation as $(x - 2)^2 + (y + 1)^2 = 5^2$, we see that this is an equation of:
  - The circle of radius 5 centered at (2, –1).



# Equation of a Circle

- So, notice that the center point also happens to be the midpoint of

  - So, by the Midpoint Formula, the center is:

$$\left(\frac{1+5}{2}, \frac{8-6}{2}\right) = (3, 1)$$

# Equation of a Circle

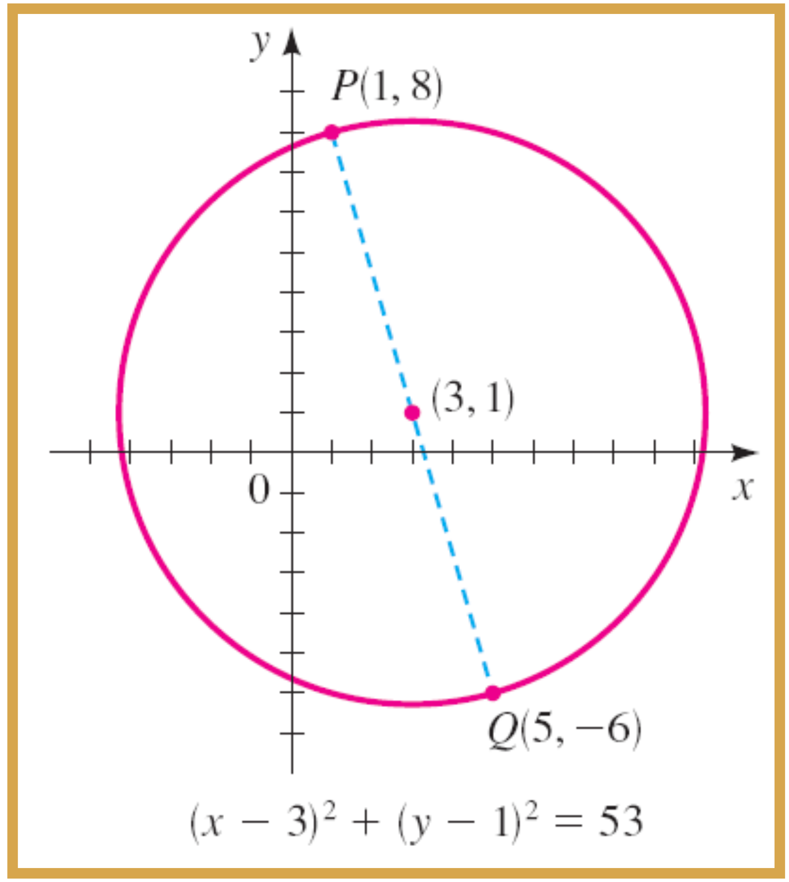
- The radius $r$ is the distance from $P$ to the center.
  - So, by the Distance Formula,

$$\begin{aligned} r^2 &= (3 - 1)^2 + (1 - 8)^2 \\ &= 2^2 + (-7)^2 \\ &= 53 \end{aligned}$$

# Equation of a Circle

- Hence, the equation of the circle is:

$$(x - 3)^2 + (y - 1)^2 = 53$$



# Equation of a Circle

- Let's expand the equation of the circle in the preceding example.

- $(x - 3)^2 + (y - 1)^2 = 53$ (Standard form)
- $x^2 - 6x + 9 + y^2 - 2y + 1 = 53$ (Expand the squares)
- $x^2 - 6x + y^2 - 2y = 43$ (Subtract 10 to get the expanded form)

# Equation of a Circle

- Notice that you can then "complete the square" to get the equation of the circle back
  - To do that, we need to know what to add to an expression like $x^2 - 6x$ to make it a perfect square.
  - That is, we need to complete the square—as in the next example.

# E.g. 7—Identifying an Equation of a Circle

- Show that the equation

$$x^2 + y^2 + 2x - 6y + 7 = 0$$

represents a circle.

- Find the center and radius of the circle.

# Identifying an Equation of a Circle

- First, notice you should have $x^2$ and $y^2$
- (big indicators)
- Second, group the $x$-terms and $y$-terms.
- Then, we complete the square within each grouping.
  - We complete the square for $x^2 + 2x$ by adding $(\frac{1}{2} \cdot 2)^2 = 1$.
  - We complete the square for $y^2 - 6y$ by adding $[\frac{1}{2} \cdot (-6)]^2 = 9$.

# Identifying an Equation of a Circle

$$(x^2 + 2x) + (y^2 - 6y) = -7$$

(Group terms)

$$(x^2 + 2x + 1) + (y^2 - 6y + 9) = -7 + 1 + 9$$

(Complete the square by adding 1 and 9 to each side)

$$(x + 1)^2 + (y - 3)^2 = 3$$

(Factor and simplify)

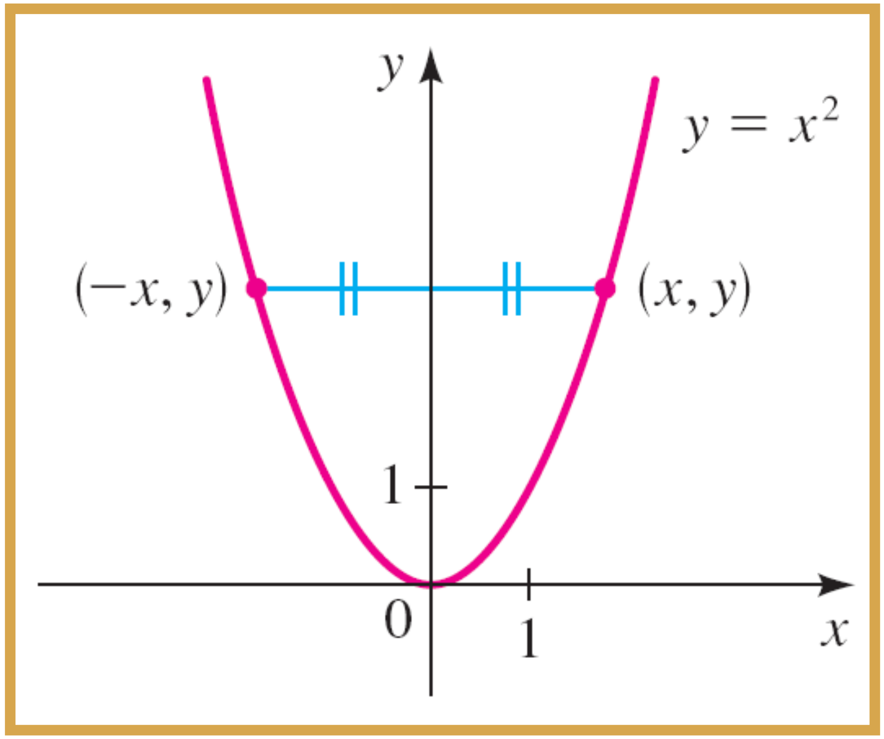
- Symmetry – an attribute of a shape or relation (function), exact reflection of form on an opposite sides of a dividing line such as axis.

# Symmetry

- The figure shows the graph of

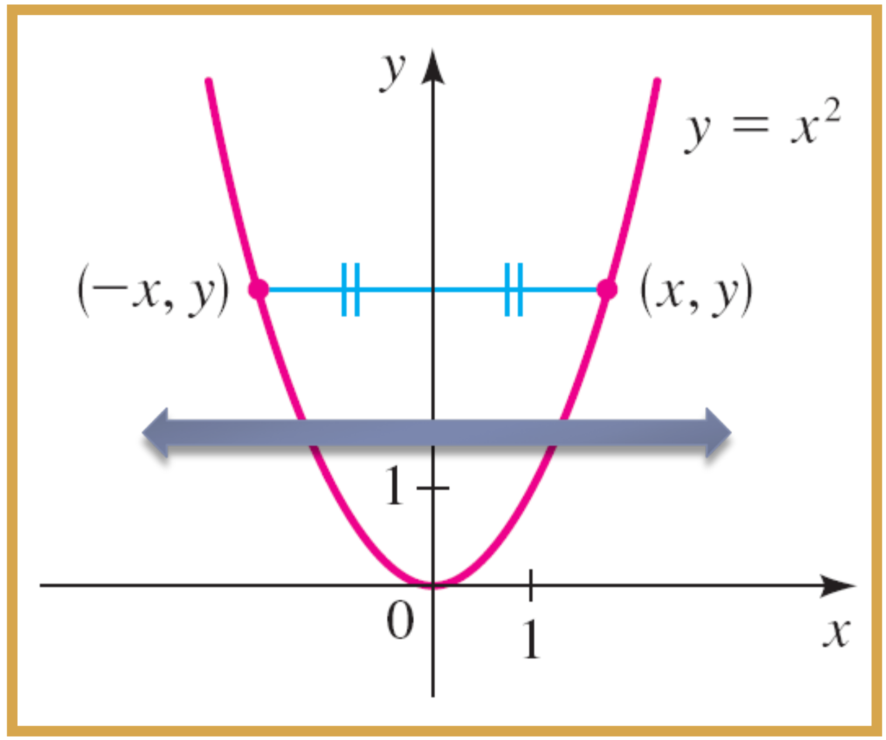
$$y = x^2$$

  - Notice that the part of the graph to the left of the *y*-axis is the mirror image of the part to the right of the *y*-axis.



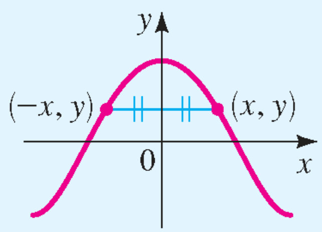
# Symmetry

- The reason is that, if the point $(x, y)$ is on the curve, then so is $(–x, y)$, and these points are reflections of each other about the $y$-axis.



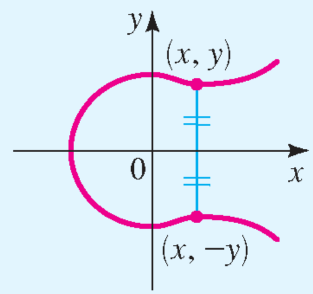
# Symmetric with Respect to y-axis

- In this situation, we say the graph is symmetric with respect to the *y*-axis.

| Type of symmetry | How to test for symmetry | What the graph looks like (figures in this section) | Geometric meaning |
|---|---|---|---|
| **Symmetry with respect to the *y*-axis** | The equation is unchanged when $x$ is replaced by $-x$ |   (Figures 2, 3, 4, 6, 10, 12) | Graph is unchanged when reflected in the *y*-axis |

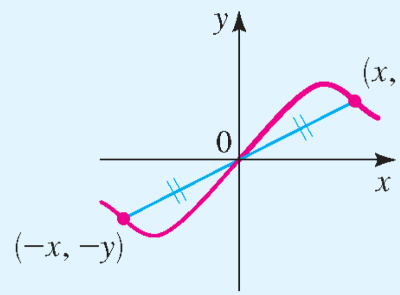
# Symmetric with Respect to x-axis

- Similarly, we say a graph is symmetric with respect to the *x*-axis if, whenever the point
  $(x, y)$ is on the graph, then so is $(x, -y)$.

| Type of symmetry | How to test for symmetry | What the graph looks like (figures in this section) | Geometric meaning |
|---|---|---|---|
| **Symmetry with respect to the *x*-axis** | The equation is unchanged when $y$ is replaced by $-y$ | <br>(Figures 6, 11, 12) | Graph is unchanged when reflected in the *x*-axis |

# Symmetric with Respect to Origin

- A graph is symmetric with respect to the origin if, whenever $(x, y)$ is on the graph, so is $(-x, -y)$.

| Type of symmetry | How to test for symmetry | What the graph looks like (figures in this section) | Geometric meaning |
|---|---|---|---|
| **Symmetry with respect to the origin** | The equation is unchanged when $x$ is replaced by $-x$ and $y$ by $-y$ |   (Figures 6, 12) | Graph is unchanged when rotated 180° about the origin |

# Using Symmetry to Sketch a Graph

- Test the equation $x = y^2$ for symmetry and sketch the graph.

•If $y$ is replaced by $-y$ in the equation $x = y^2$, we get:

- $x = (-y)^2$ (Replace $y$ by $-y$)
- $x = y^2$ (Simplify)

So, the equation is unchanged. Thus, the graph is symmetric about the $x$-axis.

# E.g. 8—Using Symmetry to Sketch a Graph

- However, changing $x$ to $-x$ gives the equation

$$-x = y^2$$

  - This is not the same as the original equation.
  - So, the graph is not symmetric about the $y$-axis.

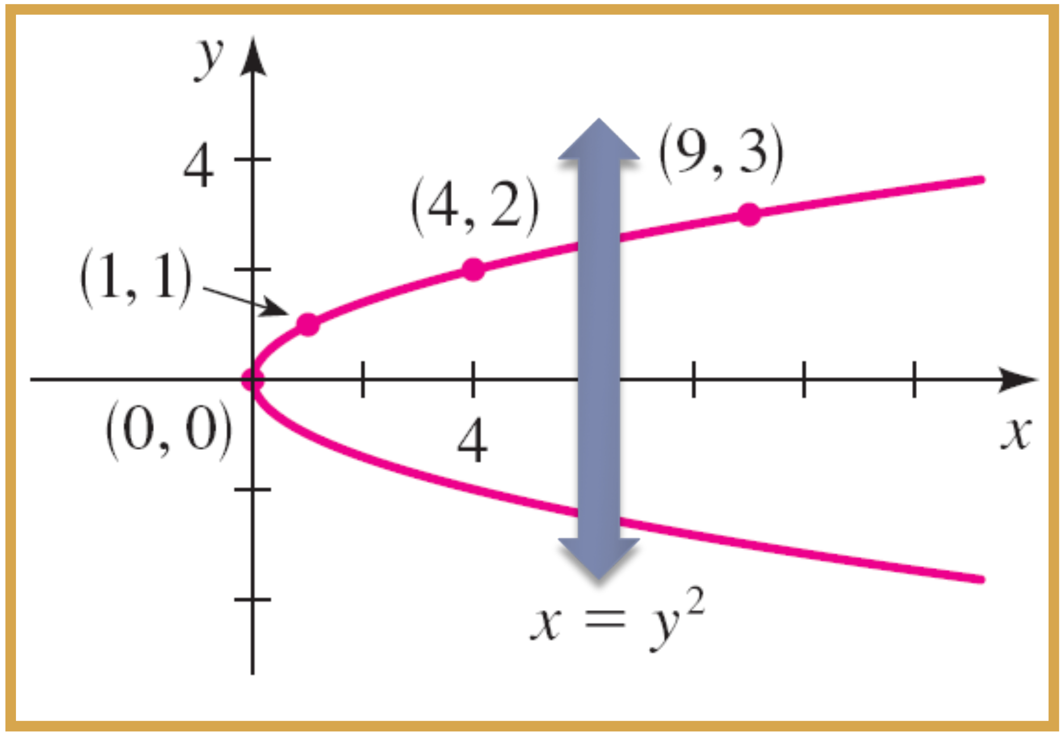
# E.g. 8—Using Symmetry to Sketch a Graph

- We use the symmetry about the *x*-axis to sketch the graph.
- First, we plot points just for $y > 0$.

| $y$ | $x = y^2$ | $(x, y)$ |
|---|---|---|
| 0 | 0 | (0, 0) |
| 1 | 1 | (1, 1) |
| 2 | 4 | (4, 2) |
| 3 | 9 | (9, 3) |

# E.g. 8—Using Symmetry to Sketch a Graph

- Then, we reflect the graph in the *x*-axis.



# Testing an Equation for Symmetry

- If we replace $x$ by $-x$ and $y$ by $-y$, we get:

$$-y = (-x^3) - 9(-x)$$
$$-y = -x^3 + 9x \quad \text{(Simplify)}$$
$$y = x^3 - 9x \quad \text{(Multiply by } -1\text{)}$$

  - So, the equation is unchanged.
  - This means that the graph is symmetric with respect to the origin.