2 Coordinates and Graphs

2.1 The Coordinate Plane

Coordinate Geometry

- "The coordinate plane is the link between algebra and geometry."
- It is a palette for plotting the expressions and equations we have been working with
  - In the coordinate plane, we can draw graphs of algebraic equations.
  - In turn, the graphs allow us to visualize the relationship between the variables in the equation.

The Coordinate Plane

- Points on a line can be identified with real numbers to form the coordinate line.

- Similarly, points in a plane can be identified with ordered pairs of numbers to form the coordinate plane or Cartesian plane.
  - (x,y) is a point on a plane, a plane is a 2D surface

Axes

- To do this, we draw two perpendicular real lines that intersect at 0 on each line.

- One line is horizontal with positive direction to the right and is called the x-axis.
- The other line is vertical with positive direction upward and is called the y-axis.

Origin & Quadrants

- The point of intersection of the *x*-axis and the *y*-axis is the origin *O*.
- The two axes divide the plane into four quadrants, labeled I, II, III, and IV here.

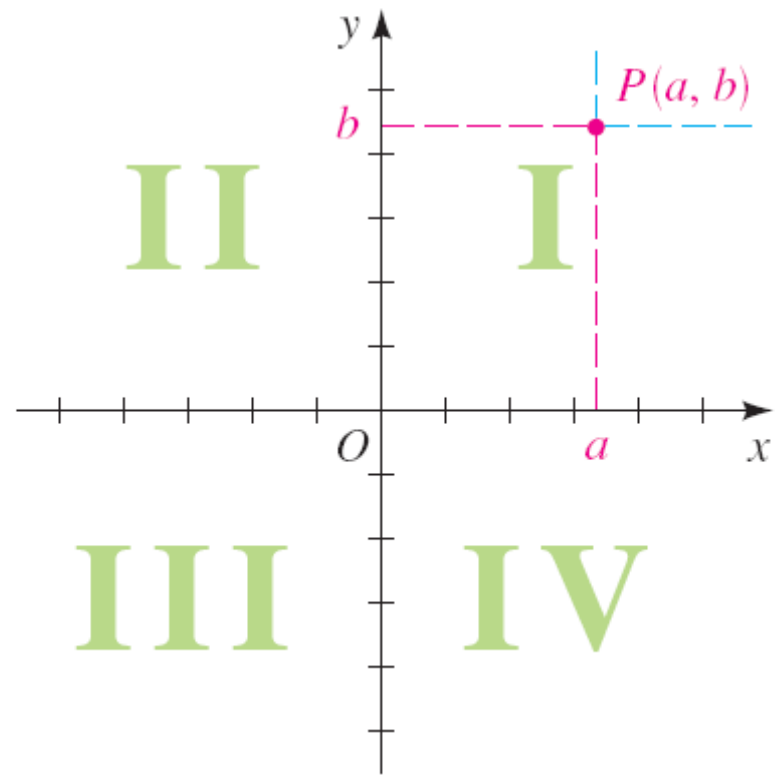


Figure 1

Origin & Quadrants

- The points on the coordinate axes are not assigned to any quadrant.

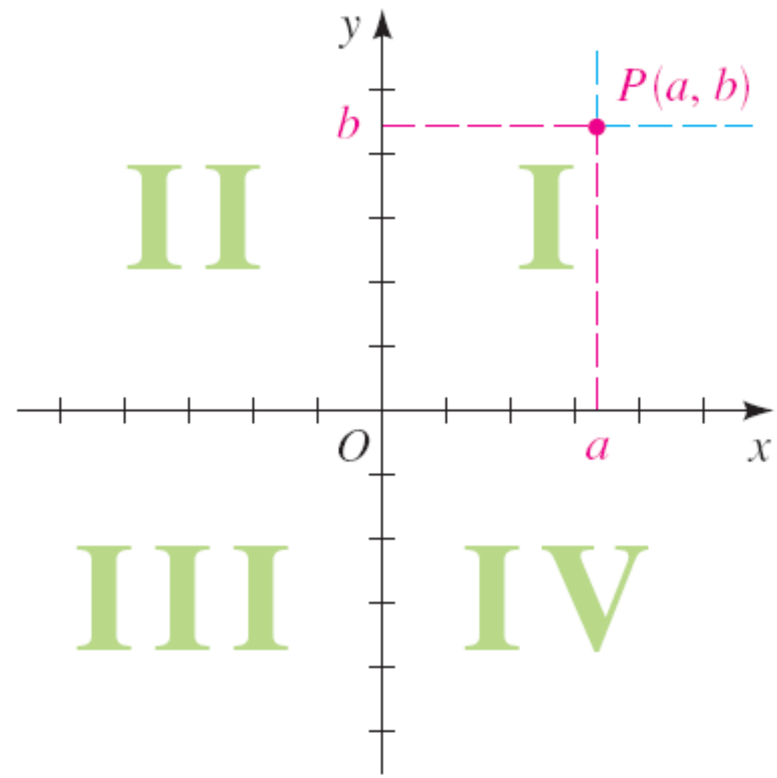


Figure 1

Ordered Pair

- Any point *P* in the coordinate plane can be located by a unique ordered pair of numbers (*a*, *b*).
  - The first number *a* is called the *x*-coordinate of *P*.
  - The second number *b* is called the *y*-coordinate of *P*.

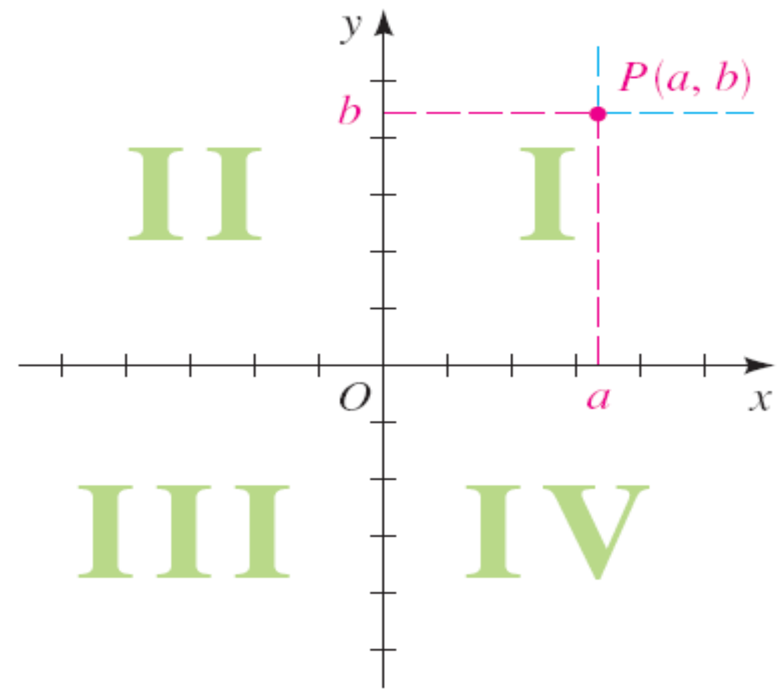


Figure 1

Coordinates

- The coordinates of a point are its address or location in the coordinate plane
  - They specify its location in the plane.

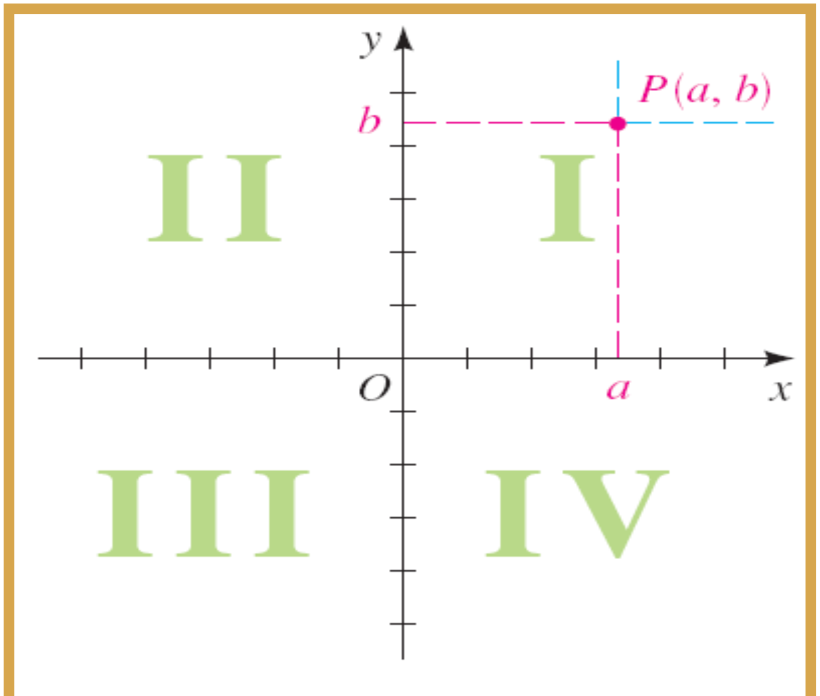


Figure 1

Coordinates

- Several points are labeled with their coordinates in this figure.

When x < 0, you will be in which quadrants?

How about when y< 0?

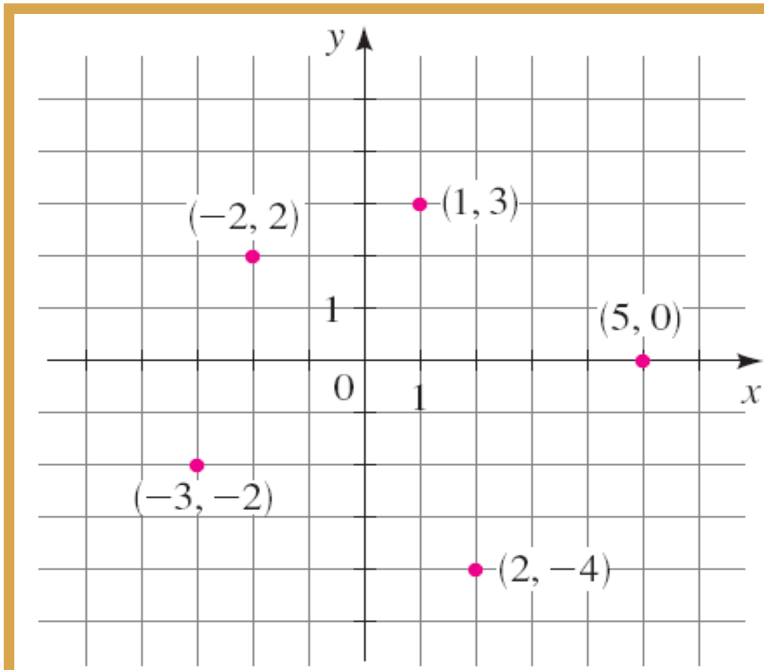


Figure 2

Graphing Regions in the Coordinate Plane

- Describe and sketch the regions given by each set.

(a) $\{(x, y) \mid x \geq 0\}$

(b) $\{(x, y) \mid y = 1\}$

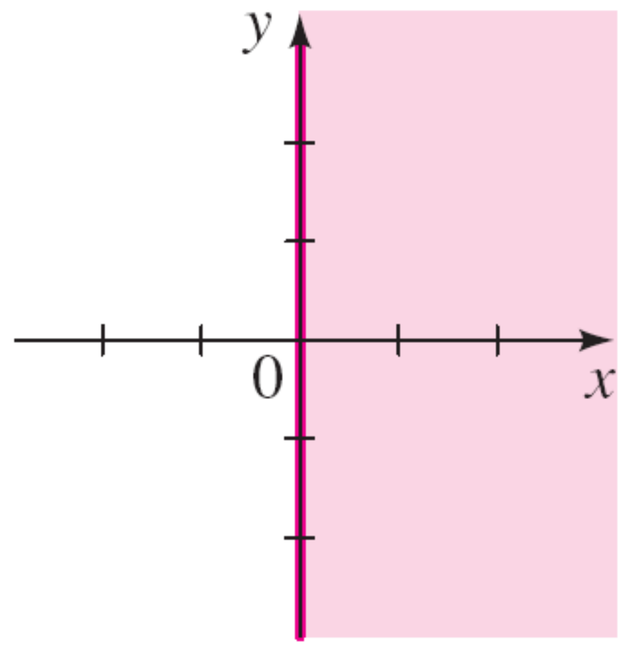
(c) $\{(x, y) \mid |y| < 1\}$

The translation is "All points of x and y such that x is greater than or equal to 0"

"|" is "such that"

Regions in the Coord. Plane

- The points whose *x*-coordinates are 0 or positive lie on the *y*-axis or to the right of it.



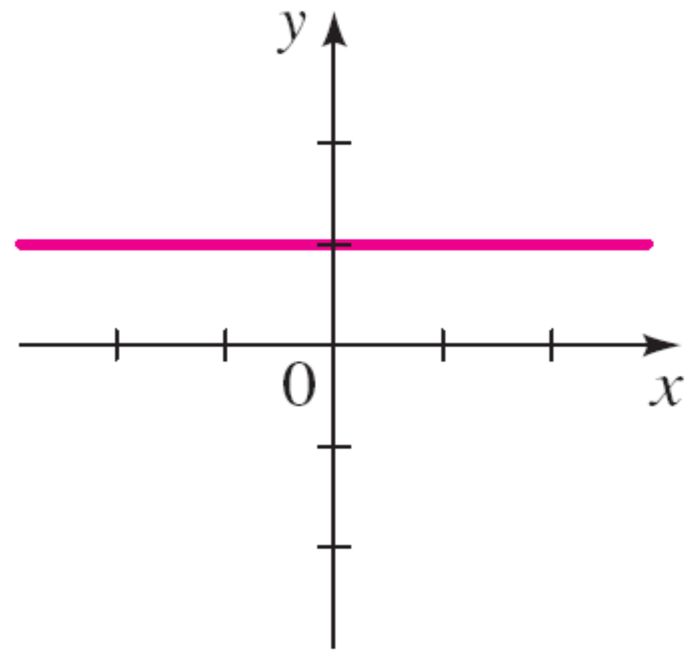
(a) $x \geq 0$

Figure 3

Regions in the Coord. Plane

- The set of all points with *y*-coordinate 1 is a horizontal line one unit above the *x*-axis.

When a variable in the coordinate plane equals a constant it is line



(b) $y = 1$

Figure 3

Regions in the Coord. Plane

- Recall from Section 1.7 that

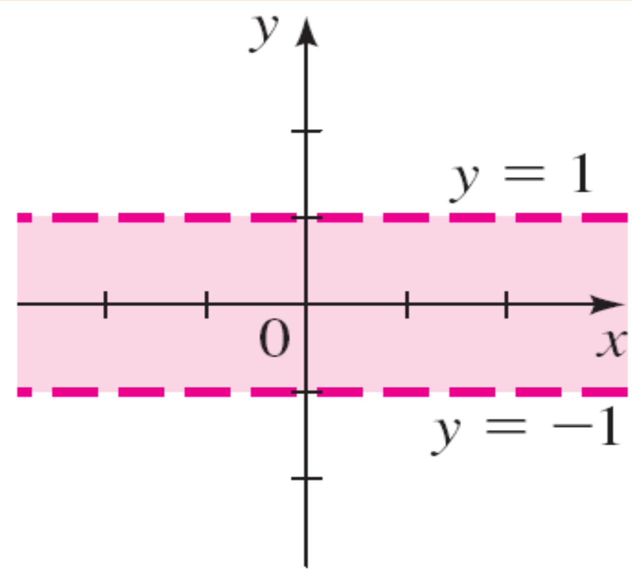
  $|y| < 1$ if and only if $-1 < y < 1$

  - So, the given region consists of those points in the plane whose y-coordinates lie between -1 and 1.
  - Thus, the region consists of all points that lie between (but not on) the horizontal lines $y = 1$ and $y = -1$.

Example (c)

E.g. 1—Regions in the Coord. Plane

- These lines are shown as broken lines here to indicate that the points on these lines do not lie in the set.
- Like the open interval concept



(c) $|y| < 1$

Figure 3

- The Distance Formula

The Distance Formula

- So, from the figure, we see that:
  - The distance between the points $A(x_1, y_1)$ and $C(x_2, y_1)$ on a horizontal line must be $|x_2 - x_1|$.
  - The distance between $B(x_2, y_2)$ and $C(x_2, y_1)$ on a vertical line must be $|y_2 - y_1|$.

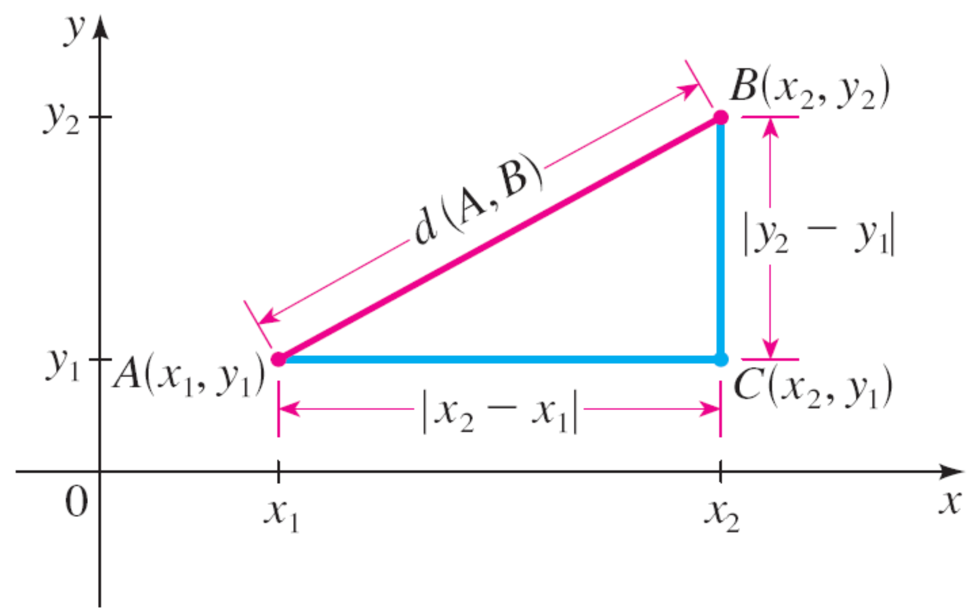


Figure 4

The Distance Formula

- Triangle *ABC* is a right triangle.
- So, the Pythagorean Theorem gives:

$$d(A,B)=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

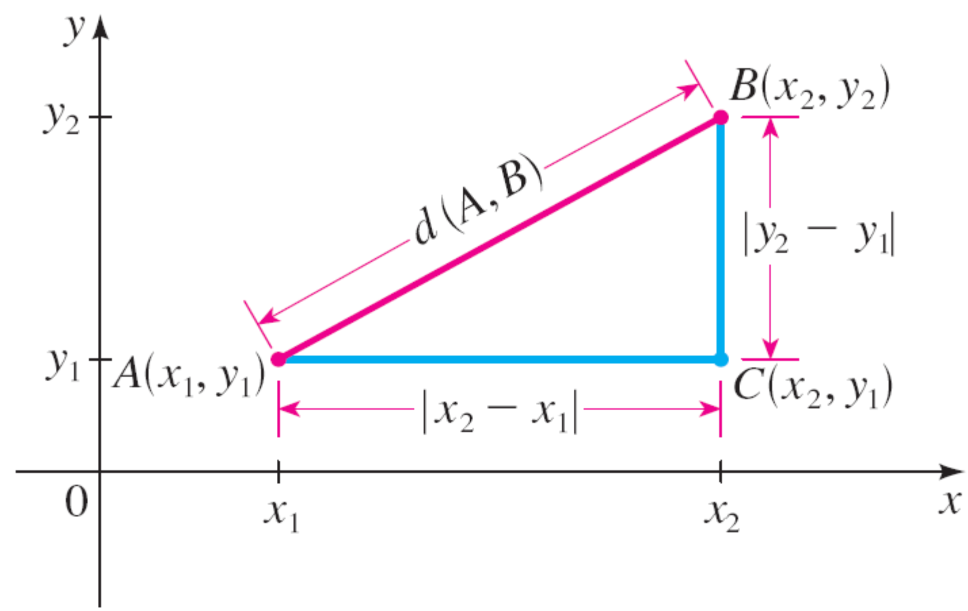


Figure 4

Distance Formula

- The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is:

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

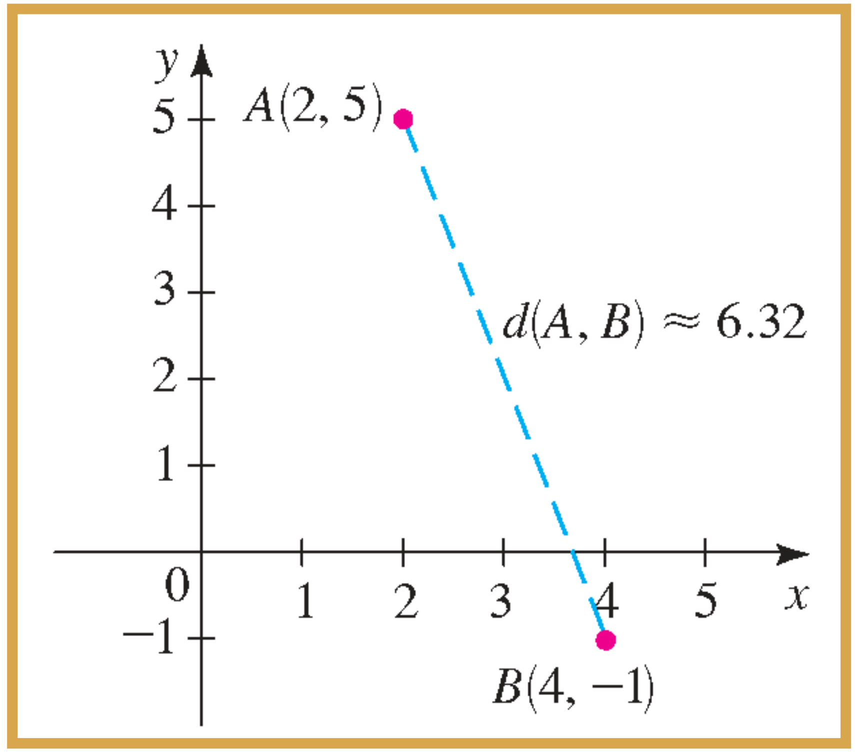
Finding the Distance Between Two Points

- Find the distance between the points $A(2, 5)$ and $B(4, -1)$.
  - Using the Distance Formula, we have:

$$
\begin{aligned}
d(A, B) &= \sqrt{(4-2)^2 + (-1-5)^2} \\
&= \sqrt{2^2 + (-6)^2} \\
&= \sqrt{4+36} \\
&= \sqrt{40} \approx 6.32
\end{aligned}
$$

Finding the Distance Between Two Points

- We see that the distance between points *A* and *B* is approximately 6.32.



E.g. 3—Applying the Distance Formula

- Which of the points $P(1, -2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$?

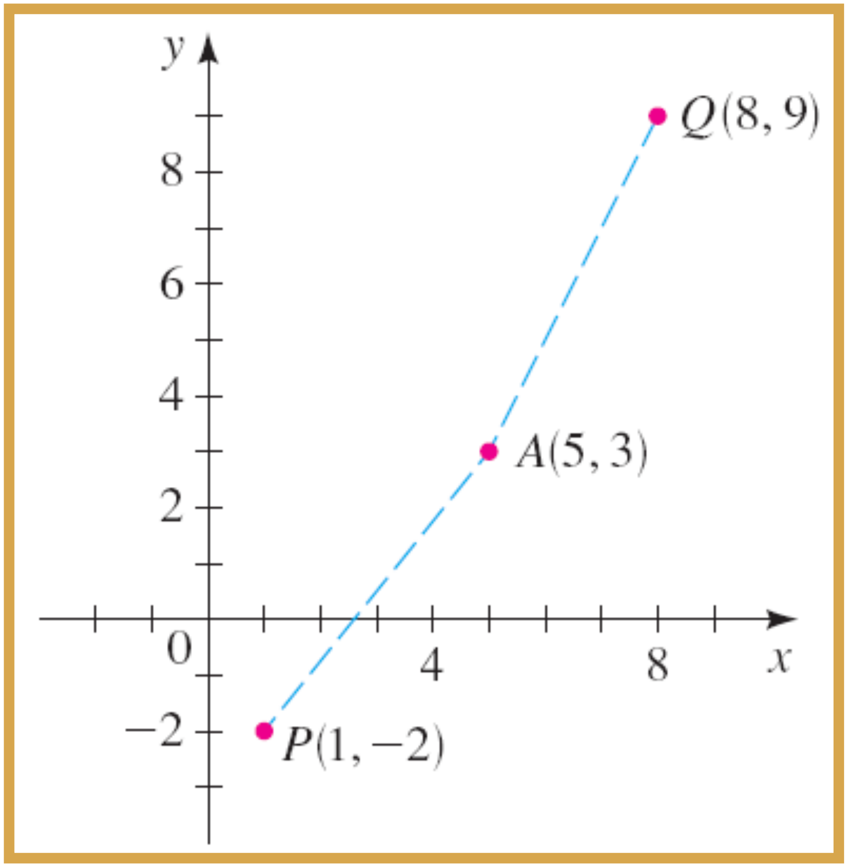
  - By the Distance Formula, we have:

$$d(P, A) = \sqrt{(5-1)^2 + [3-(-2)]^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$d(Q, A) = \sqrt{(5-8)^2 + [3-9]^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45}$$

Applying the Distance Formula

- This shows that $d(P, A) < d(Q, A)$
- So, P is closer to A.



- The Midpoint Formula

The Midpoint Formula

- Let's find the coordinates (x, y) of the midpoint M of the line segment that joins the point $A(x_1, y_1)$ to the point $B(x_2, y_2)$.
- Goal: Find a point on the line between point A and B
- What is the address of A and B?

Midpoint Formula

- The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is:

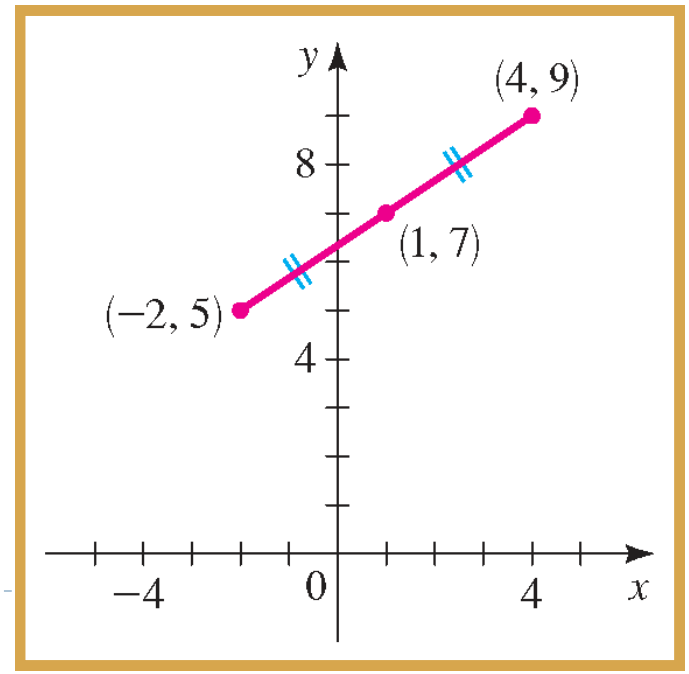
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Makes sense, if you are on a Cartesian coordinate system where the origin is 0

Finding the Midpoint of a Line

- The midpoint of the line segment that joins the points (–2, 5) and (4, 9) is

$$\left(\frac{-2+4}{2}, \frac{5+9}{2}\right) = (1, 7)$$



Applying the Midpoint Formula

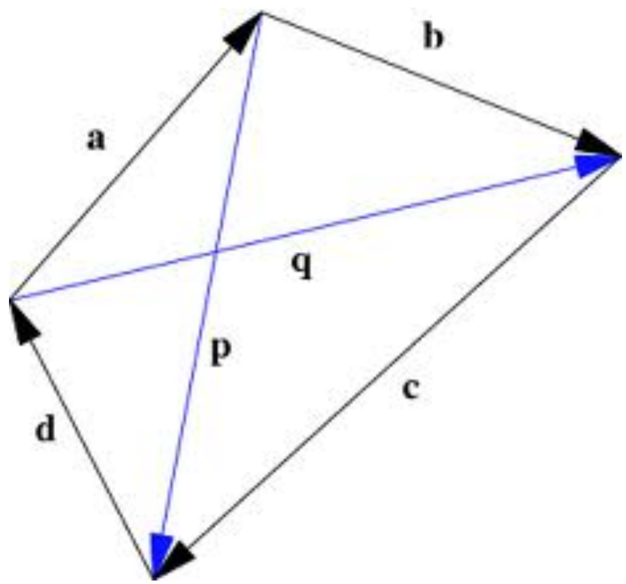
- Show that the quadrilateral with vertices

  $P(1, 2)$, $Q(4, 4)$, $R(5, 9)$, and $S(2, 7)$

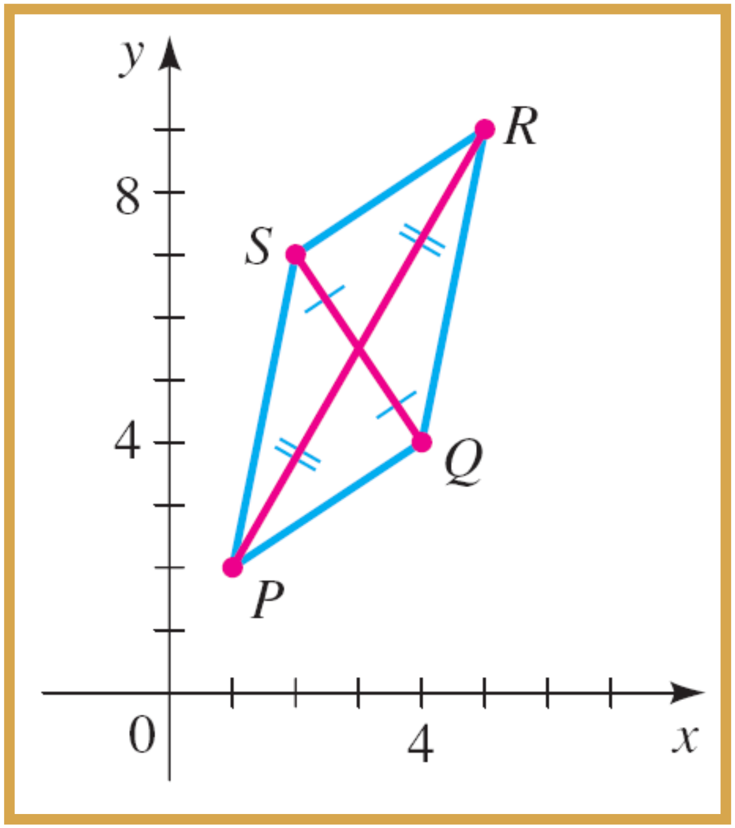
  is a parallelogram by proving that its two diagonals bisect each other.

WHAT?????

Applying the Midpoint Formula



Applying the Midpoint Formula



The corners are vertices and they are labelled points, PRSQ. (Not too original)

Each point has two coordinates

P->R is connected by a line

S->Q is connected by a line

The midpoint of both lines is the midpoint of the quadrilateral

Applying the Midpoint Formula

- If the two diagonals have the same midpoint, they must bisect each other.
  - The midpoint of the diagonal *PR* is:

$$\left(\frac{1+5}{2}, \frac{2+9}{2}\right) = \left(3, \frac{11}{2}\right)$$

  - The midpoint of the diagonal *QS* is:

$$\left(\frac{4+2}{2}, \frac{4+7}{2}\right) = \left(3, \frac{11}{2}\right)$$

Applying the Midpoint Formula

- **Thus, each diagonal bisects the other.**
  - A theorem from elementary geometry states that the quadrilateral is therefore a parallelogram.