CS310

Regular Expressions

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NFA-DFA equivalence

- Th 1.39: Every NFA has an equivalent DFA

Corollary: A language is regular if and only if there exists an NFA that recognizes it

Proof:

If the language is regular, there exists a DFA that recognizes it. Each DFA is an NFA. Conversely, if there exists an NFA that recognizes the language, convert the NFA to a DFA.

Regular Expressions

- Use regular operations (Union, Concat, Kleene Star) and languages to create a regular expression R whose *value* is a language L(R)
  - not unique in general
  - order of operations: *, concat, ∪

$R = 0^*10^*$, $L(R)=\{w \mid w \text{ has exactly one } 1\}$

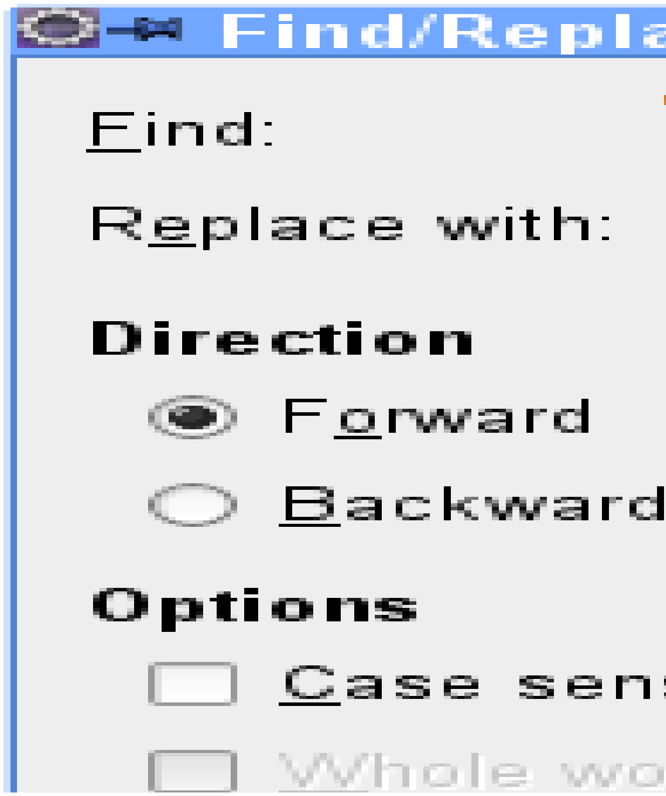
Regular Expressions

R = 0*10*, L(R)={w | }

Regular Expression libraries

```
java.util.regex //java
import re # python
<regex.h> /*GNU C library*/
```

∑ is used to represent one symbol from the language



Exercise

- {w | (w starts with 0 and has odd length) or (w starts with 1 and has even length) }

NFA?

How do we write this as a RE?

Definition

- An expression R is Regular if:

  $R = a,\ a \in \Sigma$

  $R = \varepsilon$

  $R = \emptyset$

  $R = R_1 \cup R_2$, R_1 , R_2 are regular

  $R = R_1 R_2$, R_1 , R_2 are regular

  $R = R_1^*$, R_1 is regular

- Theorem: A language is regular if and only if some regular expression describes it
  – Can be represented by an NFA

Proof

- Lemma (1.55): If L is described by a regular expression R, then there exists an NFA that accepts it

Proof: For each type of regular expression, develop an NFA that accepts it.

$R = a,\ a \in \Sigma$

$R = \varepsilon$

$R = \emptyset$

$R = R_1 \cup R_2$, R_1 , R_2 are regular

$R = R_1R_2$, R_1 , R_2 are regular

$R = R_1{}^*$, R_1 is regular

Example

- $aa^* \cup aba^*b^*$

Exercise

- {w | every odd position of w is 1 }   NFA?

How do we write this?

Exercise

- {w | w does not contain 110 }   NFA?

How do we write this?

Exercise

- {w| w contains even # 0s or exactly two 1s}

  NFA?

  How do we write this?

Proof

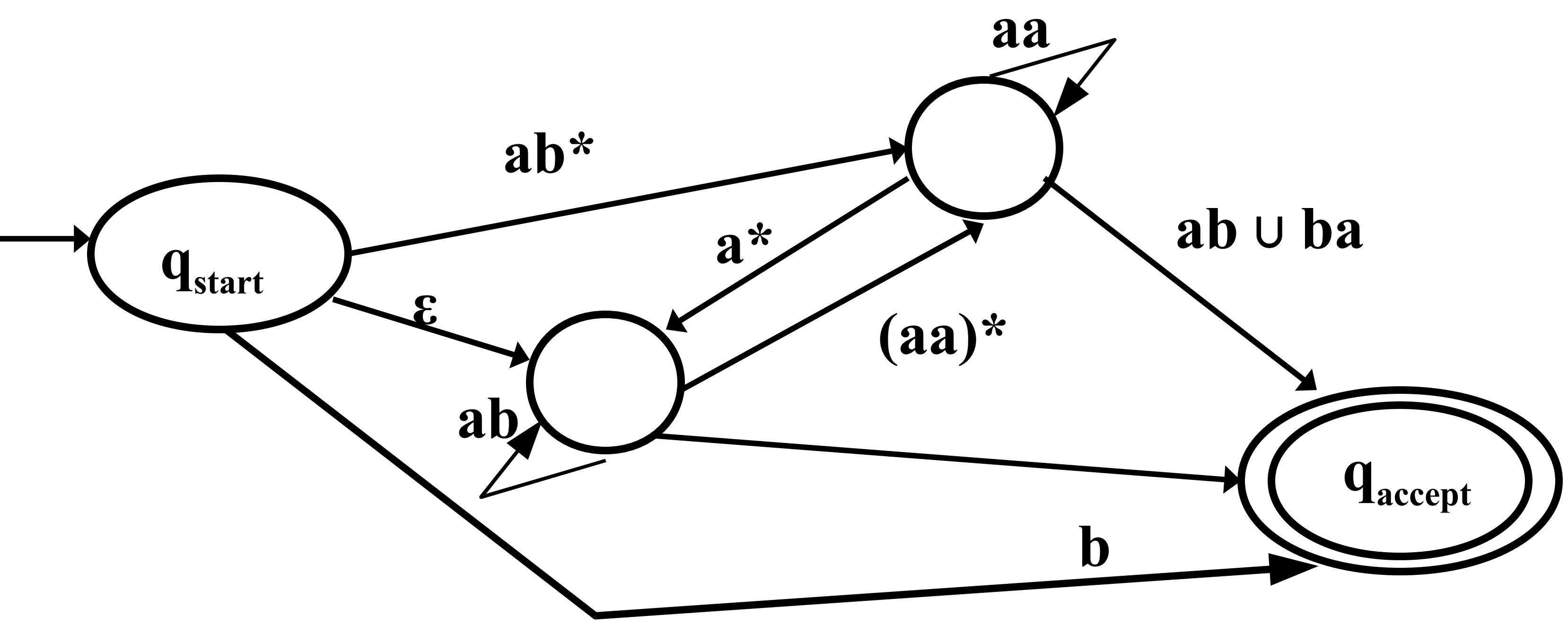
- Lemma: If a language is regular, it is described by a regular expression

- Proof Idea: If a language is regular, there exists a DFA that accepts it. We need to convert a DFA to a regular expression.

  Steps:
  - Convert DFA to GNFA
  - Convert GNFA to Regular Expression
  - GNFA?!

Generalized NFA

- NFA where the transitions may have regular expressions as labels rather than just ∑ or ε
  - Reads *blocks* of symbols from the input



  - Wait, why are we doing this?
    - to build up the regular expression slowly from the DFA

Special case of GNFA that we will use!

GNFA

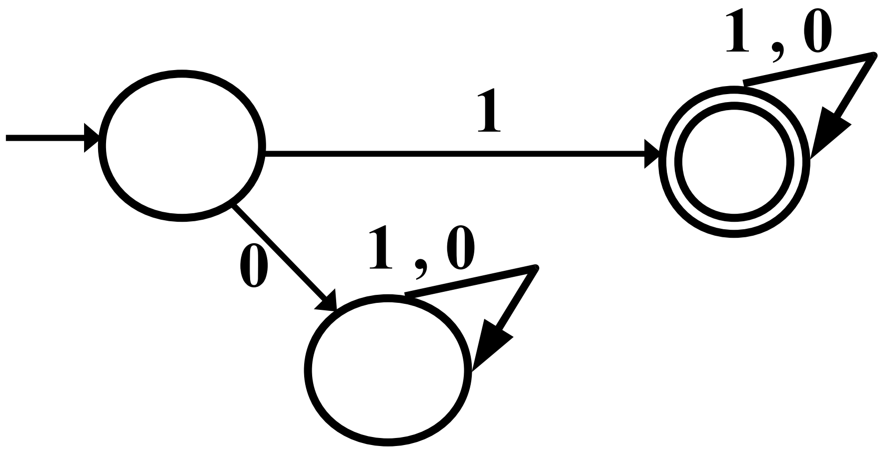
- Start state transitions to every other state, no transitions to start state
- Single accept state, transition to it from every other state, no way out, Start state != accept state
- Except for the start and accept states, one arrow goes from every state to every other state (except the start state) and also from every state to itself.

DFA to GNFA

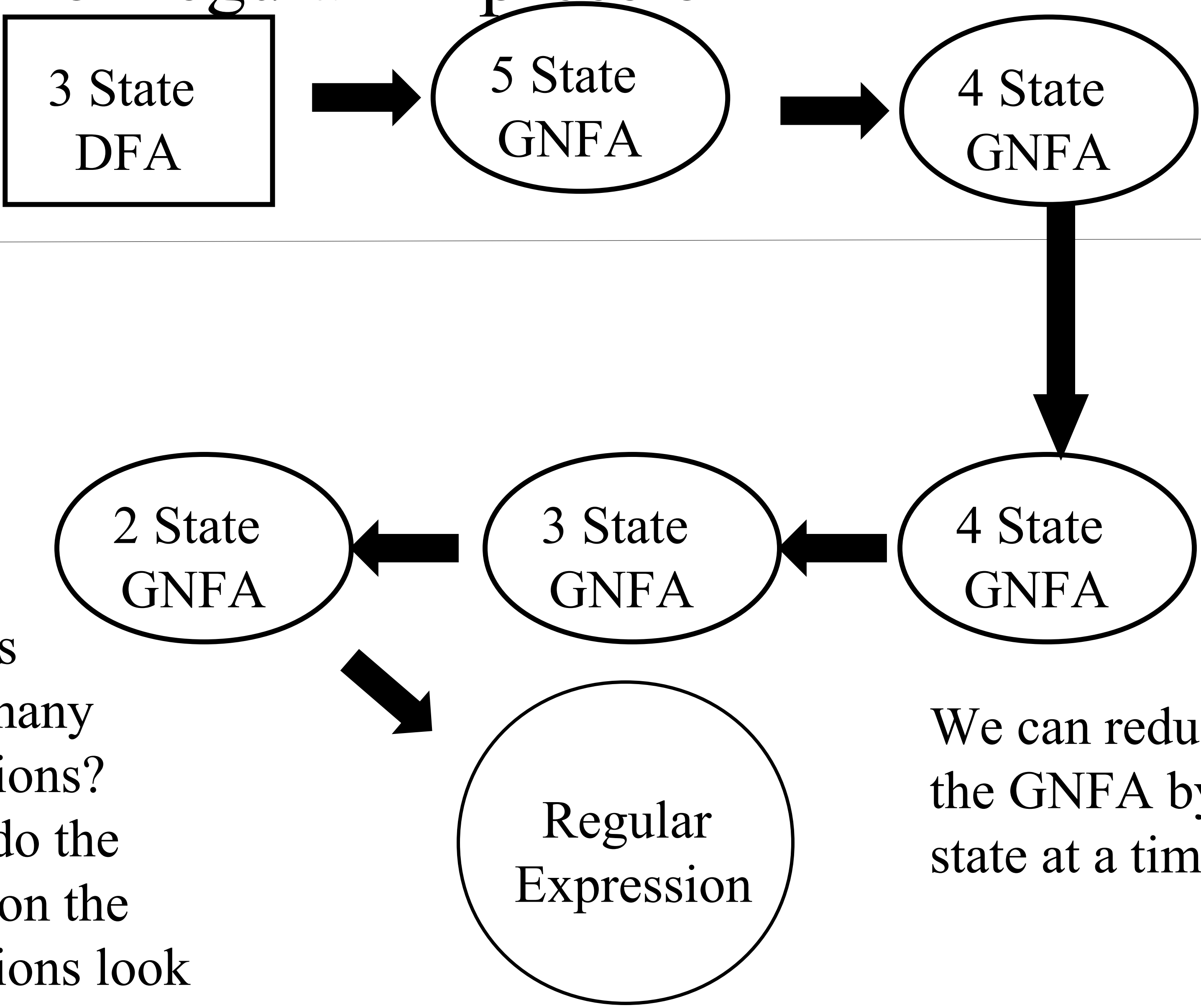
- Add new start state with ε-transitions to old start state and Ø to every other state

    Ø means you never take the transition

- Replace multiple transitions in same direction with Union
- If no transition exists between states, add transitions with Ø labels (just as placeholders)



DFA to Regular Expression



2 states
How many transitions?
What do the labels on the transitions look like?

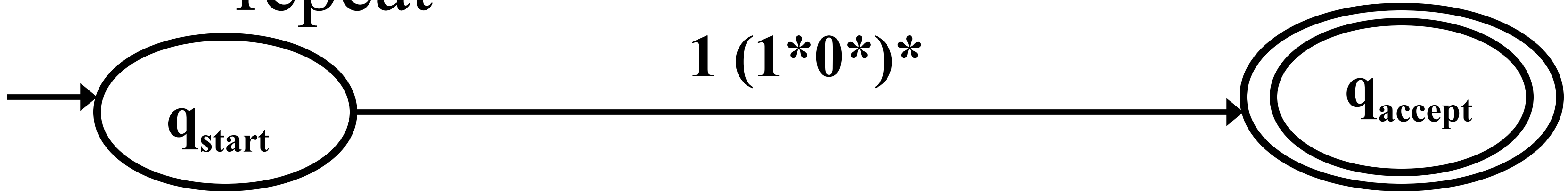
We can reduce the GNFA by one state at a time

GNFA to Regular Expression

- Each GNFA has at least 2 states (start and accept)

- To convert GNFA to Regular Expression:
  - GNFA has k states, k >= 2

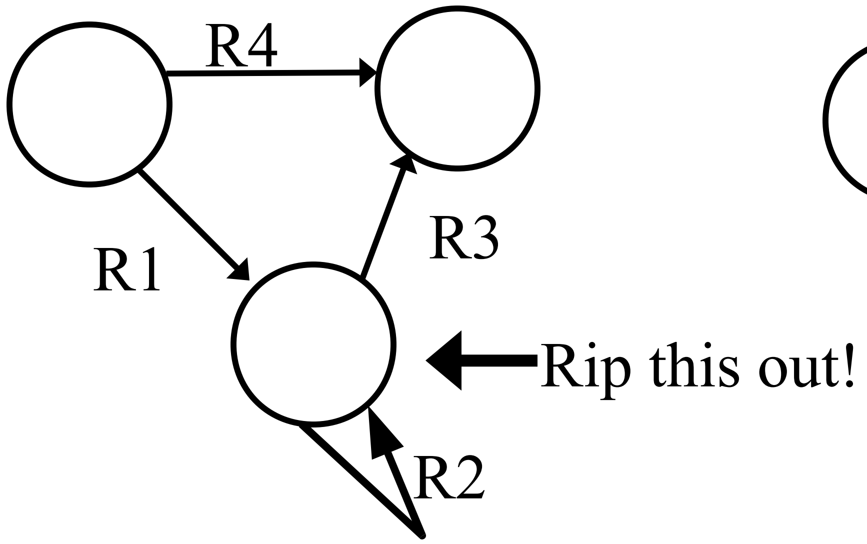
```
if k > 2 then
    Produce a GNFA with k-1 states
repeat
```

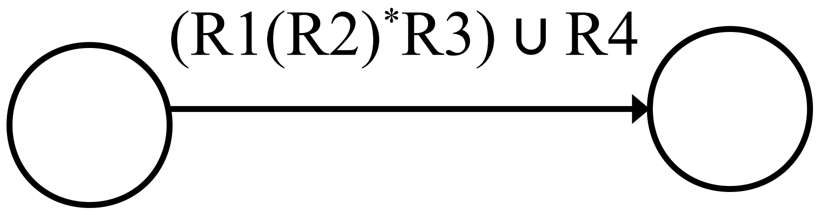

$$\begin{tikzcd} \arrow[r] & q_{start} \arrow[r, "1\ (1^*0^*)^*"] & q_{accept} \end{tikzcd}$$

GNFA to k-1 States

- Pick any state in the machine that is not the start or accept state and remove it

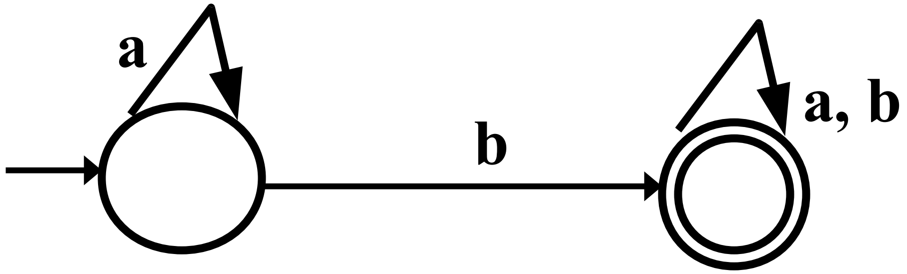
- Fix up the transitions so the language remains the same



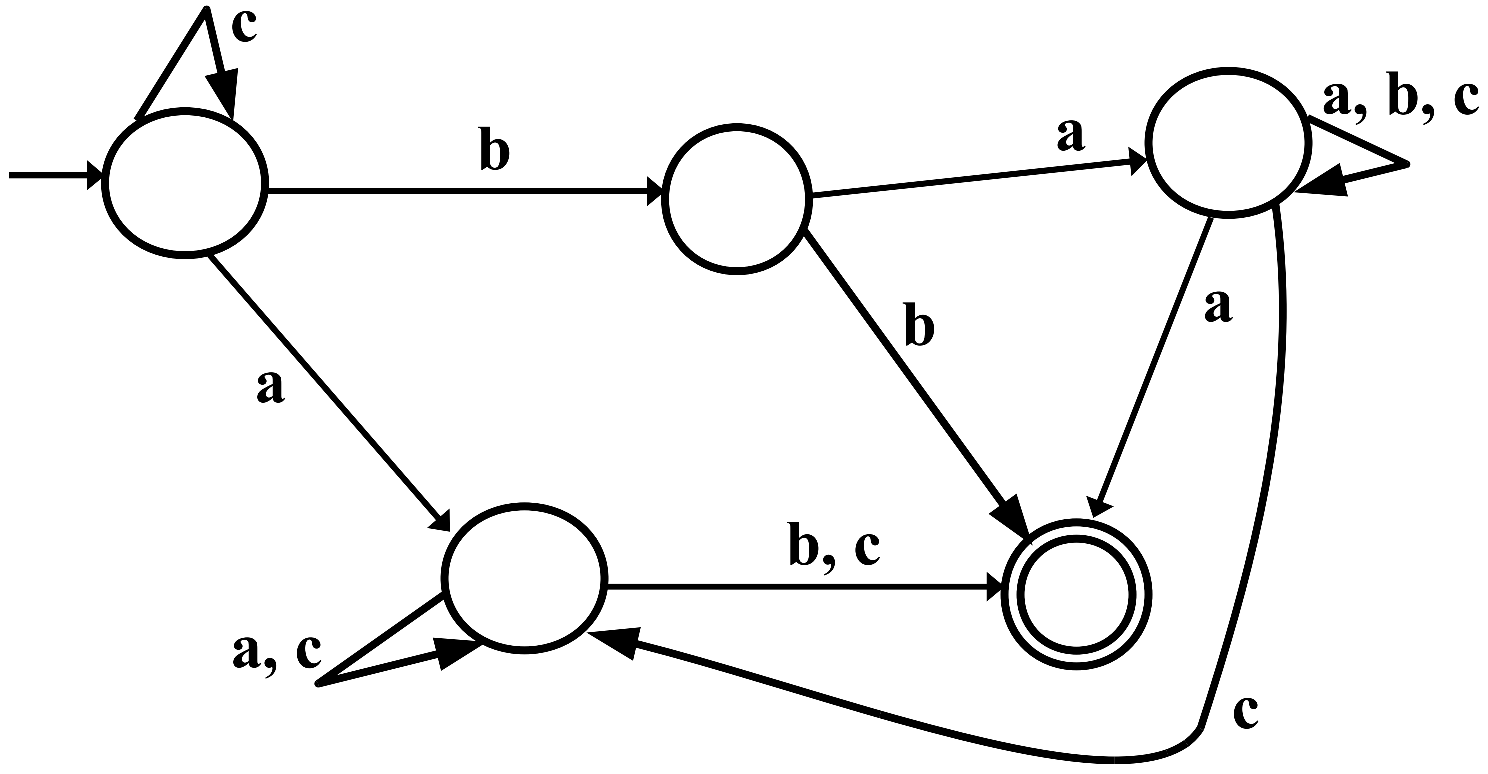


This change needs to be made for every pair of states connected through the removed state

Example, NFA to Regular Expression



Example, NFA to Regular Expression


c
b
a
a, b, c
b
a
a
b, c
a, c
c