

# CS310

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## Reducibility

Chapter 5.1

November 29, 2010

# Reduction

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- Convert one problem (A) into a second problem (B)
  - solution to B can be used to solve A
  - If B is decidable, so is A
  - If A is undecidable, so is B
  
- Is Z undecidable? Prove it is reducible to Y, which has previously been shown to be undecidable

# Halting Problem

$\text{HALT}_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ halts on input } w \}$

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undecidable?

- Proof: Assume  $\text{HALT}_{\text{TM}}$  is decidable, show that if true,  $A_{\text{TM}}$  is decidable.
  - Contradiction!
- $A_{\text{TM}}$  is reducible to  $\text{HALT}_{\text{TM}}$

- Assume TM  $R$  decides  $\text{HALT}_{\text{TM}}$
  - Use  $R$  to build TM  $S$  that decides  $A_{\text{TM}}$
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- $S$ : Run TM  $R$  on  $\langle M, w \rangle$ 
    - If  $R$  rejects, reject
    - If  $R$  accepts, run  $M$  on  $w$  until  $M$  halts
      - If  $M$  accepts, accept, if  $M$  rejects, reject
  - If  $R$  decides  $\text{HALT}_{\text{TM}}$  then  $A_{\text{TM}}$  is decidable
  - $A_{\text{TM}}$  is reducible to  $\text{HALT}_{\text{TM}}$

# TM Equality

- $EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2) \}$
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- $E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$

– undecidable (see TH 5.2 p 189)

- Show that if  $EQ_{TM}$  were decidable, so would be  $E_{TM}$

- Reduction from  $E_{TM}$  to  $EQ_{TM}$

–  $E_{TM}$  is a special case of  $EQ_{TM}$  where  $L(M_i) = \emptyset$

# Computation Histories

- List of configurations a TM goes through
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- Configuration

- Current State
- Current Tape State
- Read/Write Head location

- Finite sequence that ends in accept or reject

# Linear Bounded Automaton

- Cannot move read/write head off portion of tape with original input
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- May have larger tape alphabet than input alphabet
    - Allows for larger memory than just number of tape positions
    - Increase by constant factor

- $A_{\text{LBA}} = \{ \langle M, w \rangle \mid M \text{ is an LBA that accepts string } w \}$
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- Decidable
- Proof using computation histories
  - LBA with  $q$  states,  $g$  symbols in tape alphabet, input tape of length  $n$
  - How many possible configurations are there?
  - ???