CS310

Variants of Turing Machines

Section 3.3

November 15, 2010

Definition of Algorithm

- Algorithm: collection of simple instructions for carrying out some task (Sipser p 154)
- Hilbert: 23 mathematical problems for the new century (1900)
- 10^{th} problem: devise algorithm that tests whether a polynomial has an integral root
  - no such algorithm exists

Algorithm

- Finding an algorithm: easy
- Proving no such algorithm exists: difficult

  - especially if you can't reason about/discuss an algorithm
- 1936
  - Alonzo Church: algorithm as λ-calculus
  - Alan Turing: algorithm as machines
  - found to be equivalent
- Church-Turing Thesis
  - used to describe limits of computation
  - used on Hilbert's 10^{th} problem

Hilbert's 10th Problem

- D = { p | p is a polynomial with an integral root}

- Hilbert: Is D decidable?
  - No

- But it is Turing-recognizable
  - how could a TM recognize this language?

    $6x^3y^2 - 2x - 3y - 1$

Terminology

- We care about algorithms, not TMs

- Descriptions
  - formal
    - full states and transitions
  - implementation
    - English prose describes how the tape is used
  - high-level *
    - English prose to describe an algorithm
    - no implementation details
- Data : <G> an encoded piece of data, G
  - any data can be encoded as a string of 1s and 0s