# CS310

# Variants of Turing Machines

## Section 3.2

November 10, 2010

# Formal Definition (1 tape)

- 7-tuple
- $\{Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}\}$

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- Q: set of states
- $\Sigma$: input alphabet, not containing the blank character: ⊔
- $\Gamma$: tape alphabet, $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$
- $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function
- $q_0 \in Q$ : start state
- $q_{accept} \in Q$ : accept state
- $q_{reject} \in Q$ : reject state, $q_{accept} \neq q_{reject}$

# Multiple Tape Turing Machine

- For k tapes
  - input string is on tape 1

- Change

  $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

  to

  $\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k$

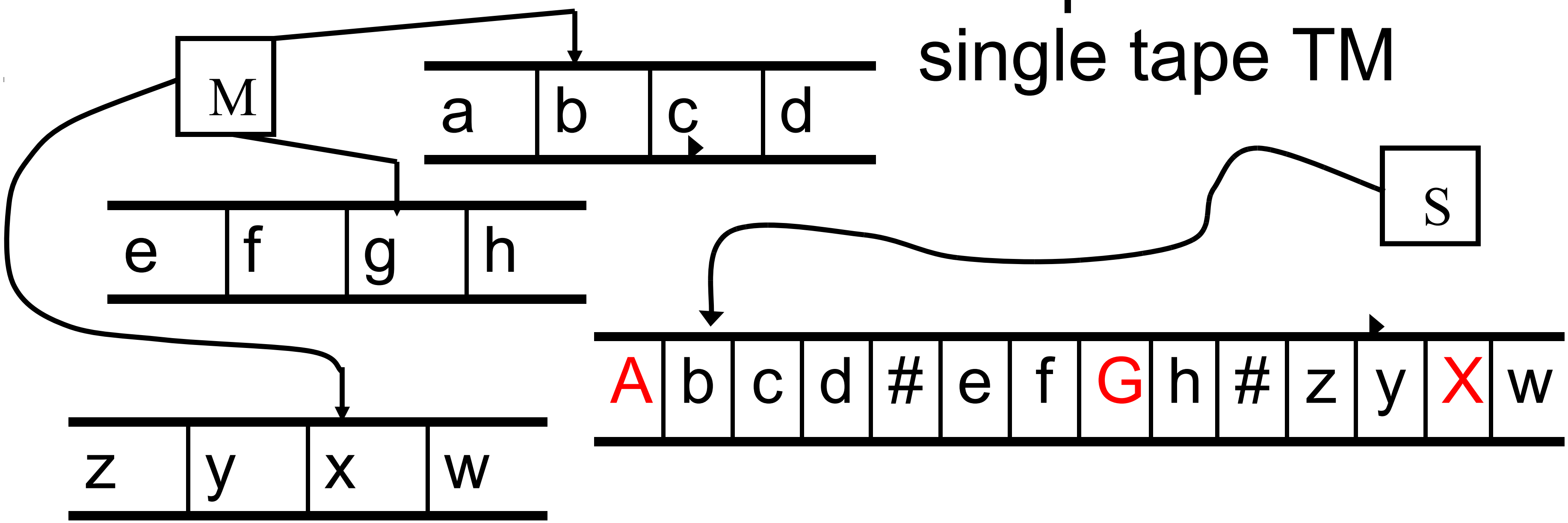
# Example

- Construct a two-tape Turing Machine to accept $L=\{a^n b^n \mid n \geq 1\}$

- Conceptually what do we want to do?

# Theorem

- Every multi-tape Turing Machine has an equivalent single tape machine
  - *adding extra tapes does not add power to the Turing Machine*
- Proof Idea: Simulate multi-tape TM as single tape TM



# Nondeterministic TM

- Just like NFA, can take multiple transitions out of a state
  - often easier to design/understand
- Design a TM to accept strings containing a *c* that is either preceded or followed by *ab*
- We can think of this computation as a tree
  - each branch from a node (state) represents one nondeterministic decision (for a single input character)

# Theorem

- Every nondeterministic TM, N, has an equivalent deterministic TM, D

- Proof Idea:
  - use a 3 tape TM (we can convert this to a one tape TM later)
  - tape 1: input tape (read-only)
  - tape 2: simulation input/output tape of the current branch of the n-d TM
  - tape 3: address tape (based on the tree) to keep track of where we are in the computation

# Practice

$\{ a^i b^j c^k \mid i > j > 0; k = 2i \}$

$\{ ww^R \mid |ww^R| \text{ is odd}, w \in \{0,1\}^* \}$

the complement of $\{ww^R \mid w \in \{0,1\}^* \}$

multiplication of two numbers in base 1:

11111 * 11 produces 1111111111