CS310

Strings, String Operators, and Languages

Sections:

September 1, 2010

Quick Review

- Sets (Union, Intersection, [Proper] Subset)

  { n | rule about n}

  Cross Product/Power Set
- Sequences/Tuples
- Functions

  $f : D \rightarrow R$
- Relation

  $f : A_1 \times A_2 \times \ldots \times A_n \rightarrow \{TRUE, FALSE\}$

  Equivalence Relations: 3 conditions

Strings

- Alphabet: Any finite set, $\sum = \{a, b\}$
- String: Any finite sequence of symbols from a given alphabet

  w = ababaabba, string over $\sum$

  ε = empty string, zero symbols

  length of w: |w| = number of symbols it contains

  $|\varepsilon| =$ $\quad |w| =$
- Strings are building blocks of computer science

  strings can represent: data sets (DNA), source code, files . . .

String Operations

- Closure (∑*): set of all strings over ∑, including ε.

  ∑ = {a, b} ∑* = {ε,a,b,ab,ba,aa,bb,…}

- Concatenations

  If x,y ϵ ∑*, then xy is defined to the be concatenation of strings x, y

  x=aba y=bab xy=

  x^k is k copies of x concatenated

  x^2 =

String Operations

- Prefix/Suffix

  $z = xy$ for $x,y,z \in \Sigma^*$, x is a prefix of z

  y is a suffix of z

- Reverse

  $x \in \Sigma^*$, x^R is the reverse of x

  $x = ab$, $x^R = ba$

Languages

- Language

  Language L over ∑ is a subset of ∑*

  L = { x ϵ {a,b}* | |x| is even }

  = {ε, aa , ab , , }

- Complement of a language L over ∑

  ∑* - L = L’

- Concatenation of languages

  L_1 and L_2 over ∑

  L_1L_2 = {xy| x ϵ L_1 , y ϵ L_2}

  L^2 = LL

Languages

- Union of languages

  L_1 and L_2 over $\sum$

  $L_1 \cup L_2 = \{x \mid x \in L_1 \text{ or } x \in L_2\}$

  L1 = {0}*

  L2 = {1}*

  what is in $L_1 \cup L_2$?

  what is in L_1L_2?

Languages

- Kleene Star

  L* = set of strings formed by concatenating any number of strings from L

  $L = \{ x \in \{ a, b\}^* \mid |x| \text{ is odd}\}$

  What does L contain:

  { }

  $L^* = \{\varepsilon, \quad , \quad , \quad , \quad \}$

Languages

- Recursive Definitions

  Define L over ∑ = {0,1} as

  1. ε є L

  2. If x є L then 0x1 є L

  What is in L? L= { }

- Can we prove that {ε,01,0011,000111,…} is equivalent to $\{0^i1^i \mid i>=0\}$?
- Show L is subset of $\{0^i1^i \mid i>=0\}$ and the reverse

Proof

- For x,y ϵ $\sum^*$, show $(xy)^R = y^R x^R$