CS310

Pumping Lemma

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September 27, 2010

Non-Regular Languages

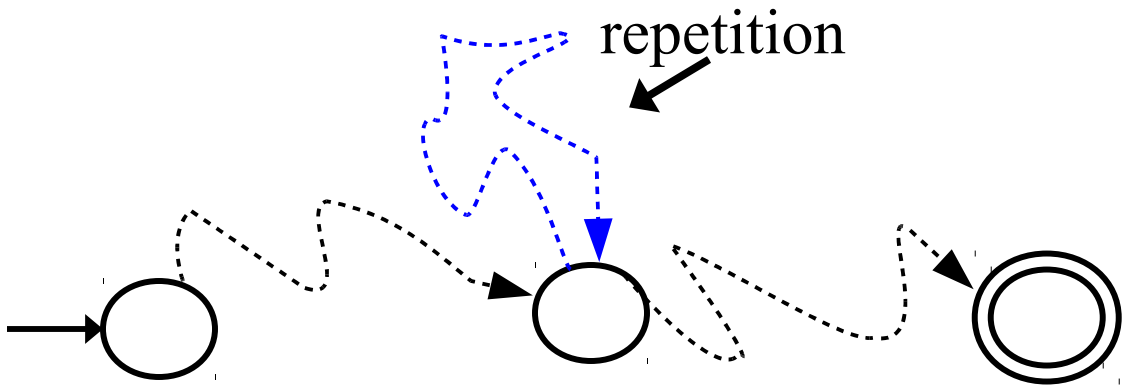
- Languages that *cannot* be represented by a finite automaton
  - Such as?
- How do we prove a language is not regular?

C = { w | w has an equal number of 0s and 1s}

D = { w | w has an equal number of occurrences of 01 and 10 as substrings }

Pumping Lemma (Informal)

Pumping: The length of the string could be 'pumped' up by repeating the cycle, *and the string would still be accepted.*



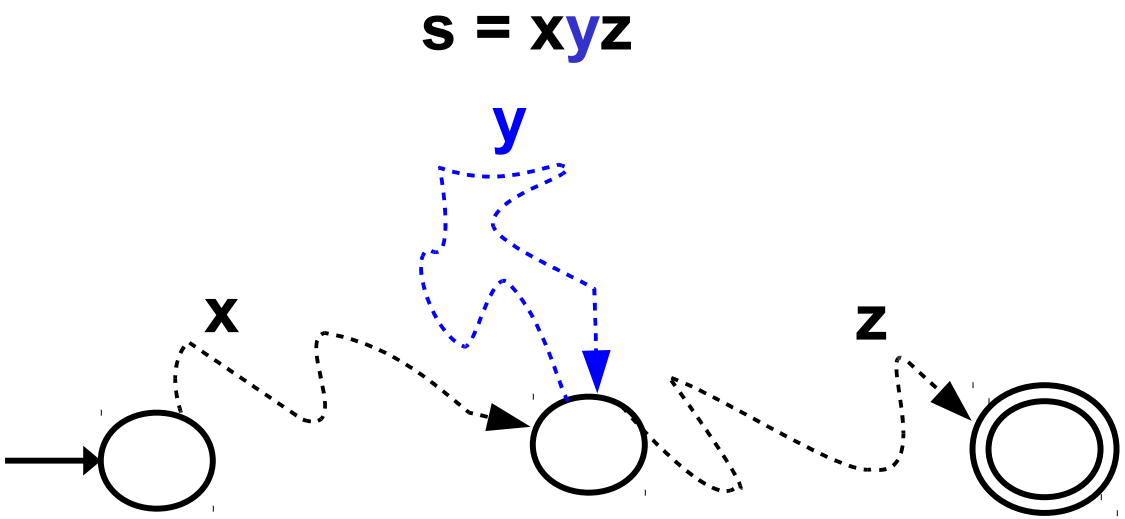
- All regular languages have a property
  - *the pumping length, p*
- |w| = n, how many states do we go through?

Pumping Lemma (Formally)

- DFA: $M = (Q, \Sigma, \delta , q_0, F)$
- If $|Q| = p$ and $s \in L(M)$ and $|s| >= p$

then there exists at least one state that was

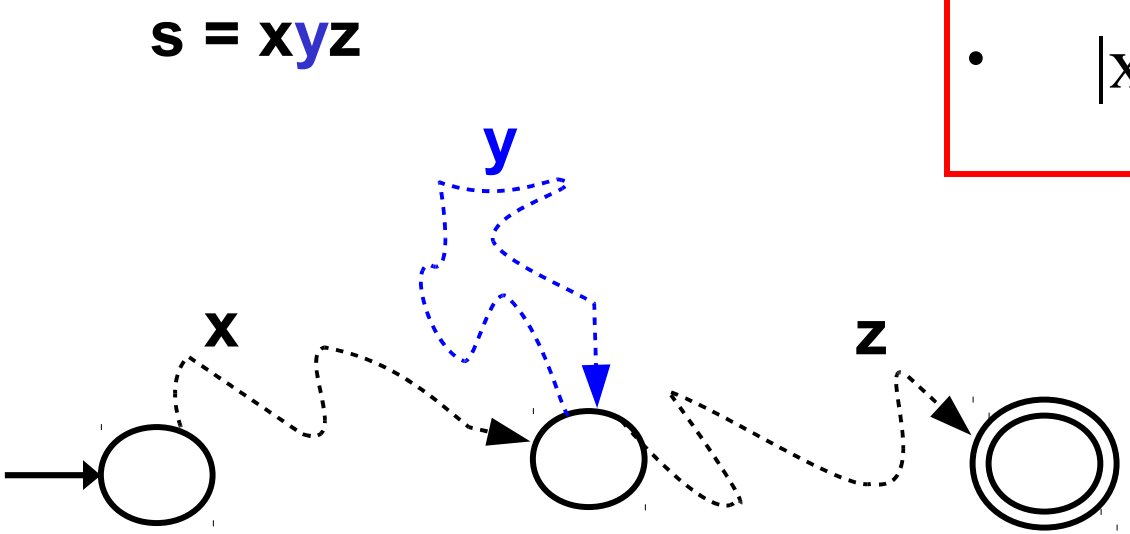
visited twice within the first *p* input symbols



Pumping Lemma (Formally)

- If A is a regular language, then :

- $i >= 0, xy^iz \in L(M)$
- $|y| > 0$ (x, z may be ε)
- $|xy| <= p$



Pumping Lemma In Action

- Find a string, $s \in L$, $|s| >= p$, that cannot be pumped to show language L is not regular.

  - Find a string that exhibits the "essence" of nonregularity
  - Proof method?

- L = { w | w contains equal number of 0s and 1s }

Practice

- $L = \{ ww \mid w \in \{0, 1\}^* \}$

  What string should we chose?

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what does ww mean?

Can that be pumped?

Regular vs Non-Regular

$L = \{ 1^* \}$ $\Sigma = \{0,1\}$

$L = \{ 1^*0^* \}$

$L = \{ 1^n \mid n \geq 0 \}$

$L = \{ 0^n1^n \mid n \geq 0 \}$

Examples Galore!

- $L = \{ a^n b^m : m > n \}$
- $L = \{ a^n b^m : m < n \}$

- $L = \{ a^n b^m : m == n \}$
- $L = \{ a^{2*n} : n > 0 \}$
- $L = \{ a^n : n \text{ is prime} \}$
- $L = \{ a^n b^m c^{n+m} : n, m > 0 \}$
- $L = \{ a^n b a^n : n >= 0 \}$
- $L = \{wbbw \mid w \in \{a, b\}^*\}$
- $L = \{ (ac)^n b^m : n > m >= 0 \}$
- $L = \{ a^n b^m : m, n > 0 \}$

Show for each language:

- Are any of these languages regular?

  Can we write any of them as a regular expression?