# Theoretical Computer Science
# CS 310

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202 Strain

Office Hours:

Mon 10:30-noon

Tues 3-4pm

Thur 1-2:30pm

and by appointment

http://zeus.cs.pacificu.edu/chadd/cs310f10/

# Syllabus

http://zeus.cs.pacificu.edu/chadd/cs310f10/syllabus.html

- *Introduction to the Theory of Computation* by Michael Sipser, (Second Edition)
  - I will assign problems out of this book
  - I expect you to do the readings

Policies:

- Class starts promptly at 2:15pm
- Assignments are due at the **beginning of class**. Late assignments will not be accepted.
- Programs that do not compile lose 70%
- The cheating policy is defined in the Pacific Catalog
- Silence all electronic devices
- Participation can raise/lower your grade

# Syllabus

**Grade Distribution**

| | |
|---|---|
| Homework | 20% |
| Unannounced Quizzes | 5% |
| Exam 1 | 25% |
| Exam 2 | 25% |
| Final | 25% |

**Percent Breakdown**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | 92-100 | A | | 90-92 | A- |
| 88-90 | B+ | | 82-88 | B | | 80-82 | B- |
| 78-80 | C+ | | 72-78 | C | | 70-72 | C- |
| 68-70 | D+ | | 60-68 | D | | | |
| 0-60 | F | | | | | | |

Tentative Dates:

- Midterm 1, Mon, October 4, 2010
- Midterm 2, Fri, November 12, 2010
- Final, Dec 4 (8:30 – 11:00 AM)

# Today

- Overview of class
- Mathematical Notation
- Proof by Induction

# Who are we?

- Is Computer Science a science?
  - Is it a *natural* science?

- What do we study?

# Overview

- What are the fundamental capabilities and limitations of computers?
- How does theory related to programming?
- Complexity Theory
- Computability Theory
- Automata Theory

# Mathematical Notation (Chap. 0)

- Basic notations we will use in this class
  - Page 16 of your book has a partial list (no symbols!)

- Set

- Subset

- Proper Subset

# Sets

- Shorthand for describing a set

  { n | rule about n}

# Set Operations

- What can we do with sets?
- Union
- Intersection
- Complement

# Sets

- Power Set

  { 0, 1 }

- Cartesian Product (Cross Product)

  { 0, 1 }  X { a, b }

# Sequences/Tuples

- Sequence

- Tuple

  K-tuple

# Functions

- Object that takes input, produces output

  f (a) = b

- Domain and Range

  $f : D \rightarrow R$

- Onto

# Functions

- $f : A_1 \times A_2 \times \ldots \times A_k \rightarrow R$

  $(a_1, a_2, \ldots, a_k)$

  k-ary

  arity

  unary (k=1) binary (k=2)

- Notation

  Infix notation: a + b

  Prefix notation: add(a,b)

# Relations

- Predicate (property)

  $f : D \rightarrow$ {TRUE, FALSE}

- Relation

  $f : A_1 \times A_2 \times \ldots \times A_n \rightarrow$ {TRUE, FALSE}

- Notation

  table

  Set

# Equivalence Relations

binary relation

shows that two objects are equal

must satisfy 3 conditions:

1. R is ***reflexive*** if for every x, xRx;
2. R is ***symmetric*** if for every x and y, xRy if and only if yRx;
3. R is ***transitive*** if for every x, y, and z, xRy and yRz implies xRz

# Proof by Contradiction

- Assume it is false
- Show this leads to a false consequence
- Prove $\sqrt{2}$ is irrational
  - Assume it is rational: $\sqrt{2} = m/n$
  - Reduce m/n to lowest terms: m and n are not both even (could reduce out a 2)
  - sometimes tricky to pick exactly what false consequence to show

# Proof by Induction

- Basis

  Prove P(1) is true

- Induction Step

  Prove that for each $i \geq 1$, if P(i) is true,

  then so is P(i+1); *assume P(i) is true*

- Basis + Induction Step

  P(1) is true, i = 1

  P( (i+1) ) is true

  P( (i+1) +1 ) is true …

# Proof by Induction

- Prove: $1 + 2 + \ldots + n = n(n+1) / 2$

  for $n \geq 1$

  Basis:

  Induction:

# Prove by Induction

- $n^2 >= 3n$, $n>=3$

- $F_n$ is the *n*th Fibonacci number, $n>=1$.
  - $F_n = F_{n-1} + F_{n-2}$
  - $F_1=1$ ; $F_2=1$

- $F_{3n}$ is even, $n>=1$
- $F_{4n}$ is evenly divisible by 3, $n>=1$

# Graphs

- Graph

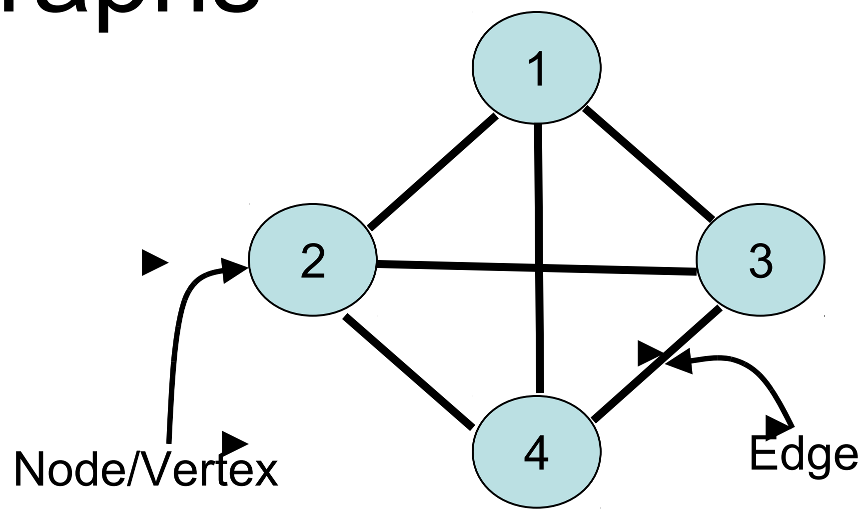
  Collection of nodes

  and edges

  G = (V,E)

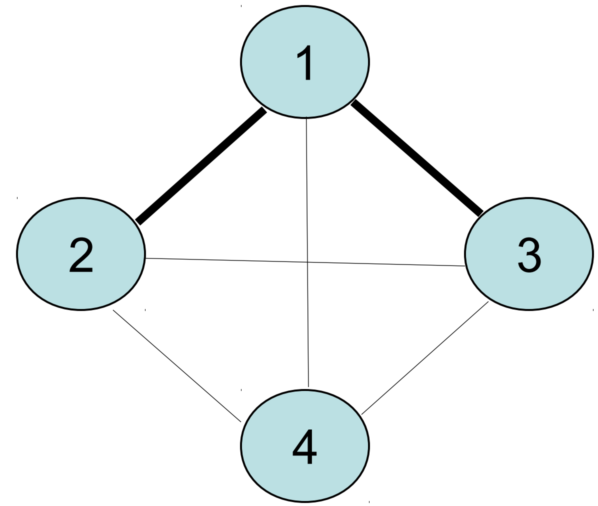
  E =

  undirected




- Degree of a node

# Graphs



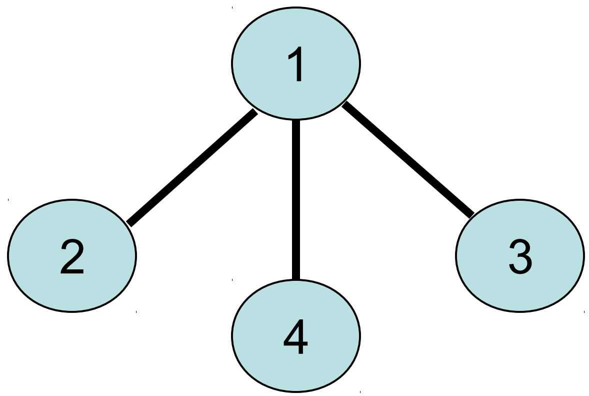

- Subgraph
- Path

Connected graph

- Cycle

# Trees

- Connected graph with no simple cycles
- Leaves
- Root



# Directed Graph

- Arrows denote which way an edge goes

  outdegree/indegree

  G=(V,E)

  V=

  E=

  Directed path

  Strongly connected