CS310

P vs NP

the steel cage death match

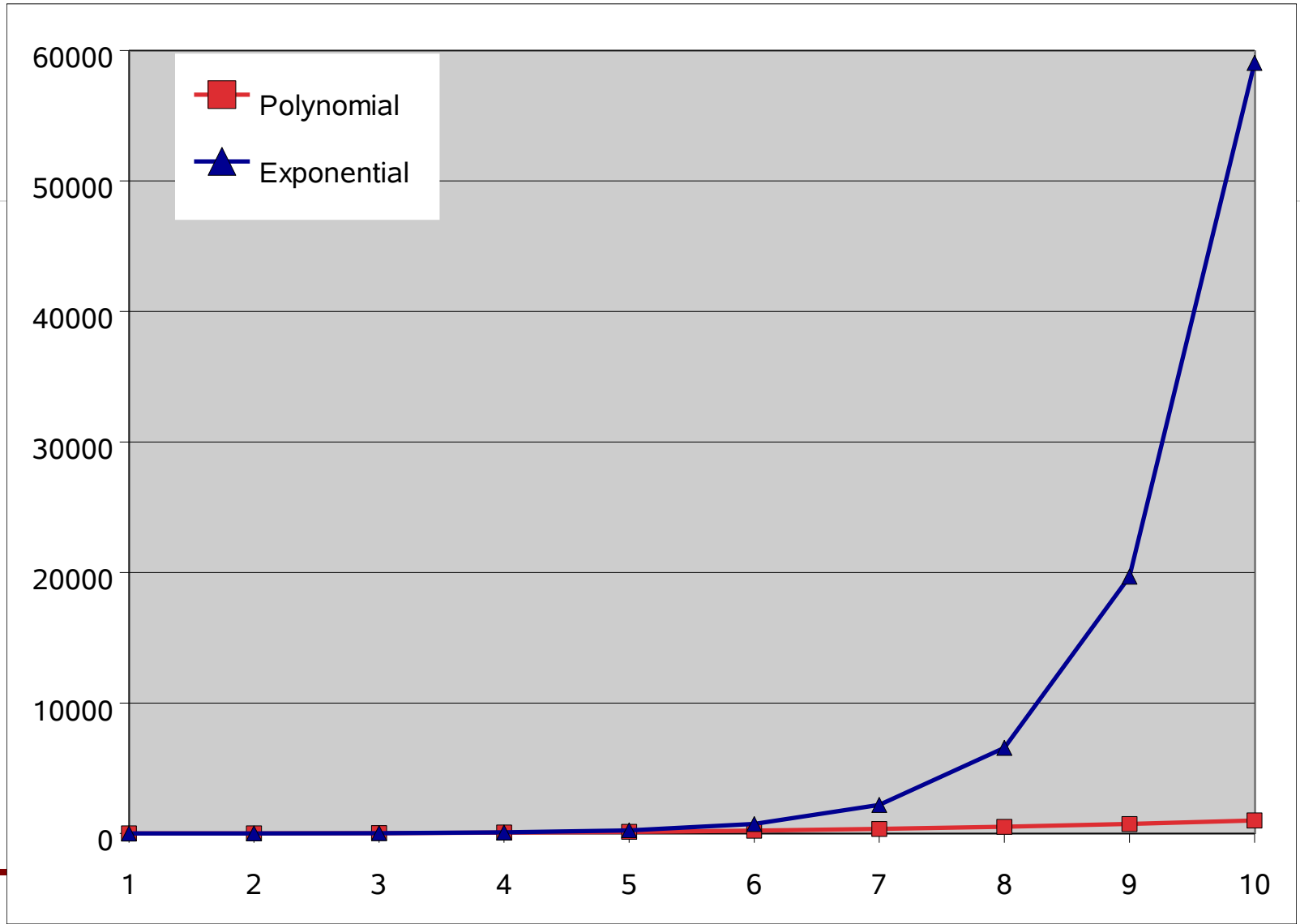
How hard is a problem to solve?

Section 7.2

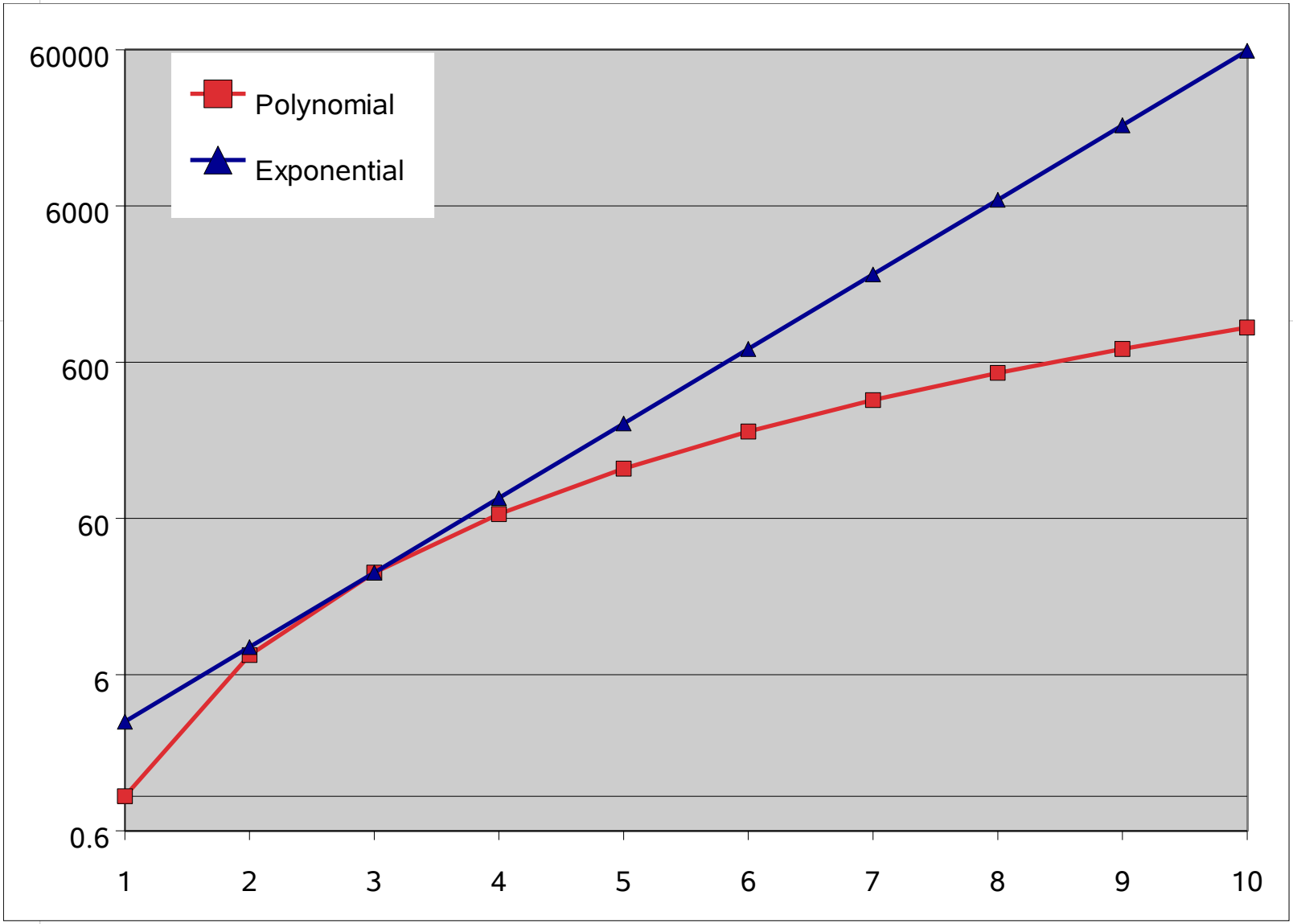
November 24, 2008

Polynomial vs Exponential

- Polynomial: n^3 Exponential: 3^n



Log Scale



Complexity relationships between models

- Theorem 7.8: let t(n) >= n, every t(n) time multi-tape TM has an equivalent $O((t(n)^2)$ time single-tape TM.
  - polynomial difference
- Theorem 7.9: Every t(n) >= n time ND single tape TM has an equivalent $2^{O(t(n))}$ time deterministic single tape TM
  - exponential difference

The class P

- P is the class of languages that are decidable in polynomial time on a deterministic, single tape TM
- Problems in class P
  - PATH: { <G,s,d> | G is a directed graph, find a directed path from s to d }
  - RELPRIME: {<x, y> | x and y are relatively prime}
    - Euclidean algorithm
- Every context-free language is in P

Real Life

- Problems in class P are usually manageable on a real computer
  - n^K
  - though k=100 may introduce some practical problems

The class NP

- NP is the class of languages that are decidable in polynomial time on a nondeterministic single tape TM
  - Problems in class NP
    - HAMPATH: { <G, s, t> | G is a directed graph, with Hamilton path from s to t } (a path that passes through every vertex of a graph exactly once)
  - These problems are decidable on a deterministic single tape TM in exponential time

Verifier

- A verifier of a language, A, is an algorithm, V, such that

  A = { w | V accepts <w, c> for some string c}

  where c is a certificate

  |c| is polynomial in terms of |w|

- NP is the class of languages that have polynomial time (in terms of the length of w) verifiers

P vs NP

- $P \subseteq NP$
  - unknown if the classes are unequal
- If P = NP, then all problems in NP can be solved in polynomial time, if we are clever enough to find the right algorithm
- NP-Completeness
  - set of problems in NP whose complexity is related to all problems in NP
  - if an NP-Complete problem can be shown to be in P, then P=NP
  - boolean satisfiability, for example