Pumping Lemma Revisited

- Is { w | w contains and equal number of As and Bs } regular?
- What MUST you do to answer this question?

CS310

Turing Machines

Section 3.1

November 3, 2008

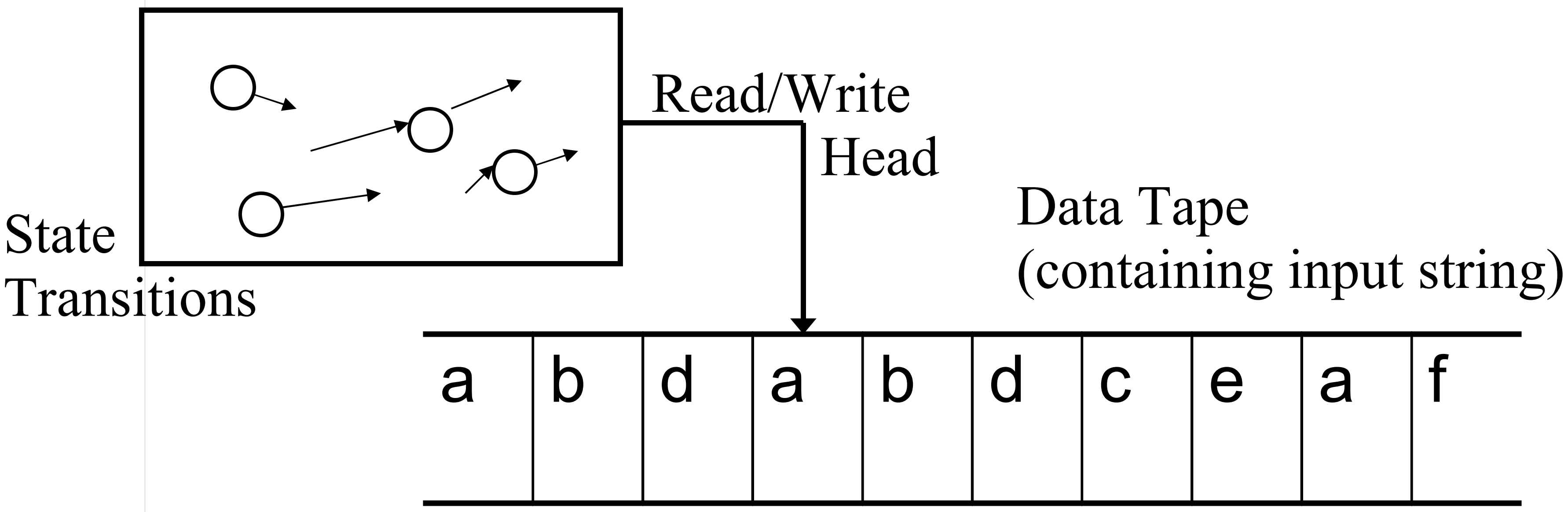
Turing Machines

- Similar to Finite Automata
  - unlimited and unrestricted memory
    - random access
  - more accurate model of modern computer
- Problems that cannot be solved by a Turing Machine cannot be solved by a "real" digital computer
  - theoretical limits of computation

> What are the fundamental capabilities and limitations of computers?
> Computer Science is really the science of computation, not of computers.

Turing Machine

- State Transitions plus infinite "data tape"
  - read tape
  - write tape
  - move around on tape



Differences with FA

- TM can read and write from tape
  - FA can only read
- Read/Write head can move left or right
  - FA can only move one direction

- TM tape is infinite
- TM accept and reject states take effect *immediately*

Church-Turing Thesis

- Turing Model is and always will be the most powerful model
  - it can simulate other models: D/NFA, PDA
  - variations do no improve it
    - extra tape
    - nondeterminism
    - extra read/write heads

Formal Definition (7 Tuple)

- $\{Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}\}$
- Q: set of states
- Σ: input alphabet, not containing the blank character: $\vee$

Γ: tape alphabet, $\vee \in \Gamma$ and $\Sigma \subseteq \Gamma$

δ: $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$: transition function

$q_0 \in Q$: start state

$q_{accept} \in Q$: accept state

$q_{reject} \in Q$: reject state, $q_{accept} \neq q_{reject}$

Operation

- Start configuration of M on input w is:q_0w
- Accepting configuration: q_{accept}
- Rejecting configuration: q_{reject}

- Yield: uaq_ibv yields uq_nacv

  if $\delta(q_i, b) = (q_n, c, L)$
- Accepting and Rejecting configurations are called *halting* configurations
  - the TM stops operating
  - otherwise, loops forever

Definition of Computing

- A TM, M, accepts a string, w, if there exists a sequence of configurations, $c_0, c_1, \ldots, c_n$, such that:
  - c_0 is the start configuration
  - c_i yields c_{i+1} for all i
  - c_n is an accept configuration
- The set of strings M accepts is L(M)
  - language of M

Notes

- Deterministic
- May make multiple passes over input

- Reject string by entering reject configuration or looping forever
  - hard to tell if a machine will loop forever
  - halting problem

Example

- L = { w#w | w $\in$ { 0,1} * }
- Conceptually, we want to do what?
- input string:

1	1	0	0	#	1	1	0	0	$\vee$

- Configuration of the TM:

  u q_n v

  u, v $\in \Gamma^*$ and $q_n \in Q$

  the read/write head is on the first character of v and the TM is in state q_n

Definitions

- Turing recognizable
  - a language is Turing Recognizable if some TM recognizes it

- Turing decidable
  - a language is Turing decidable if some TM decides it
  - *halts* on rejected strings rather than looping forever
    - hard to tell if a looping machine is really going to reject the string

Example

- L = {ΣΣ0 | Σ = {0 ,1} }

Example

- $L = \{a^n b^n \mid n >= 0\}$

Example TM

- $L = \{ w \mid |w| \text{ is even} \}$, $\Sigma = \{a, b\}$

Transducer

- A machine that produces output
- A function F with domain D is Turing-Computable if there exists a TM, M, such that the configuration q_0w yields q_{accept}, F(w) for all w $\in$ D.
- x = number in base 1, F(x) = 2x

x = 111 2x = 111111

Transducer

- x, y positive integers in base 1
- design TM that computes x+y

Transducer

- x, y positive integers in base 1, x > y
- design TM that computes x-y