

# CS310

## Non-Context-Free Languages

Sections: 2.3 page 123

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# Pumping Lemma (take two)

Theorem: For any CFG there is an equivalent grammar in CNF.

Pumping lemma (CFG): Suppose  $A$  is a CFG. There exists a number  $p$  such that

if  $s \in A$  and  $|s| \geq p$

then  $s = uxyz$  where

$ud^i xy^i z \in A, i \geq 0$

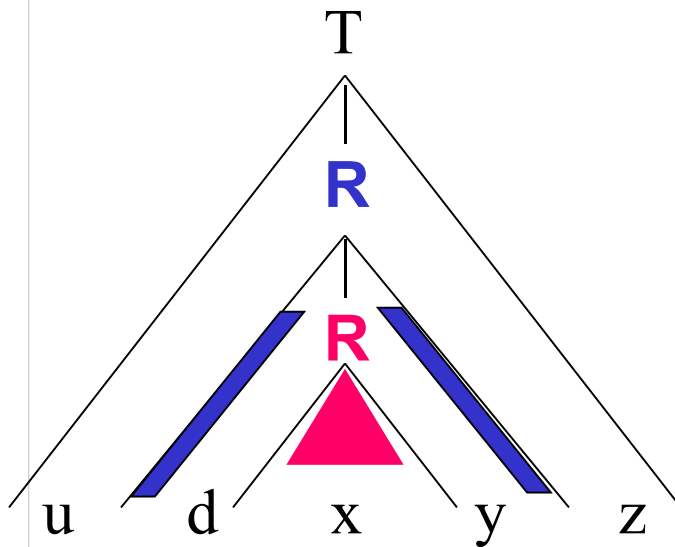
$|dy| > 0$

$|dxy| \leq p$

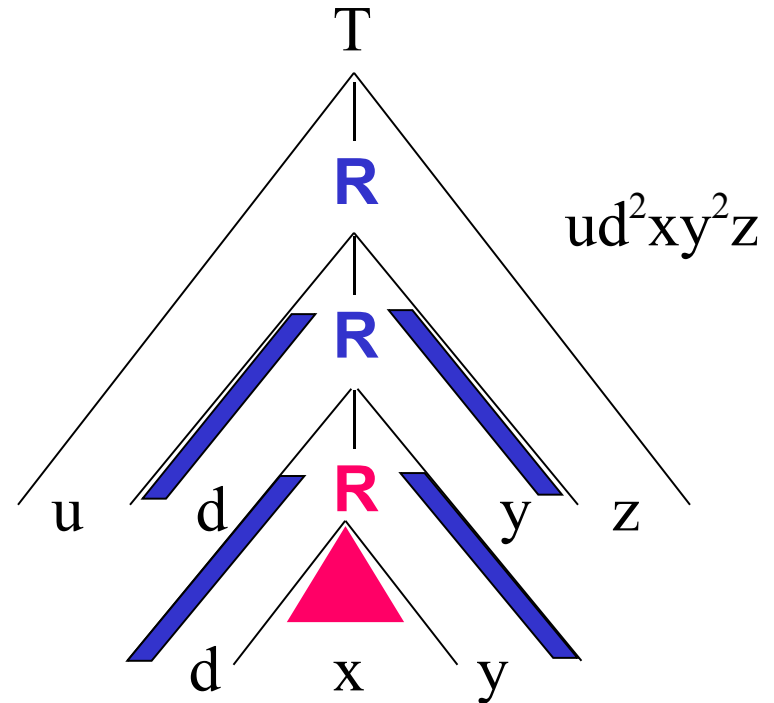
Note: Your book  
uses  $uvxyz$

But, capital  $V$  is the set of  
variables for the grammar  
so that can get confusing

# Pumping a Parse Tree



$ud^1xy^1z$



$ud^2xy^2z$

# Proof

Suppose  $A$  is a CFG in CNF and  $s \in A$ ,

$$|s| \geq p = 2^{|V|+1}$$

2 ?

The height of the parse tree for  $s$  is ?

# Example

$$L = \{a^i b^i c^i \mid i \geq 0\}$$

a PDA cannot represent this. Why?

Pumping Lemma:

s =

u =

d =

x =

y =

z =

# Example

$$L = \{a^i b^j c^k \mid k \geq j \geq i \geq 0\}$$

a PDA cannot represent this. Why?

Pumping Lemma:

s =

u =

d =

x =

y =

z =

# Example

$$L = \{ ww \mid w \in \{0, 1\}^* \}$$

$S =$

# Example

$L = \{ w \# x \mid w^R \text{ is substring of } x; w, x \in \{0, 1\}^* \}$