CS310

Pushdown Automata

Sections: 2.2

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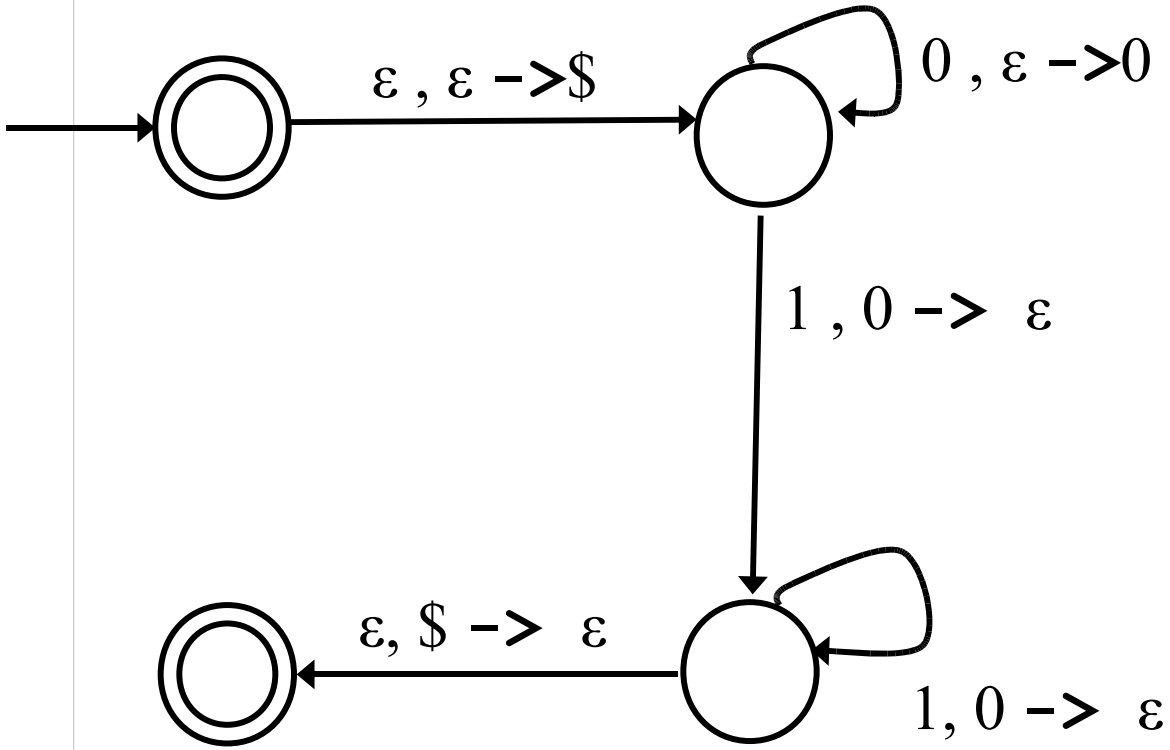
Quick Review

a , b ->c

Read a from input, read b from stack, push c onto stack to take this transition

a = ε, read no input
b = ε, don't pop data from stack
c = ε, don't push data onto stack



Example

- $L = \{ a^i b^j c^k \mid i, j, k \geq 0, i = j \text{ or } i = k \}$
  - Hint: push **a**s onto the stack

Theorem

- A Language is context free if and only if there exists a PDA that recognizes it.
- Lemma:
  - If a language is context free, then some PDA recognizes it
  - Show: a CFG can be transformed into a PDA
- Lemma:
  - If a PDA recognizes a language, then it is context free

Construct PDA from CFG

- $L = \{a^n b b^n \mid n \geq 0\}$ CFG?

1) Place $, start variable on stack

2) Repeat:
   a) if variable A is on top of stack, use replacement rule A *(pop)* ->w *(push)*
   b) if terminal on top, read input, compare. If match, repeat, else die
   c) if $ on top, enter accept, die if there's more input

Chomsky Normal Form

- CNF presents a grammar in a standard, simplified form:

  A -> BC

  A -> *a*

  S -> ε

  - Where A,B,C are variables and B and C are not the start variable
  - *a* is a terminal
  - The rule S -> ε is allowed so the language can generate the empty string (optional)

CNF Benefits

- Easier to prove statements about CFG's when in CNF
- Any CFG can be converted to CNF

- Remove productions:

  A -> ε to empty

  A -> B Unit rule

  A -> s, s contains a terminal and |s| > 1

  A -> s, |s| > 2

  $s \in \{ V \cup \sum \}^*$

Removing A -> ε

S -> UAV

A -> ε

- A variable A is ***nullable*** if A -*> ε

  Find all nullable variables

  Remove all ε transitions

  If T -> $s_1 A s_2$ and A nullable

  then add T -> $s_1 s_2$

Example

S -> TU

T -> AB

A -> aA | ε

B -> bB | ε

U -> ccA | B

Nullable variables?

Productions removed?

Productions added?

Removing A -> B (Unit Productions)

A -> B

B -> s

$s \in \{ V \cup \sum \}^*$

- A variable B is A-derivable if A -*> B

  Find all A-derivable variables for each A

  Remove all unit transitions

  If B -> s and B is A-derivable

  then add A -> s

Example

S -> TU | T | U

T -> AB | A | B

A -> aA | a

B -> bB | b

U -> ccA | B | cc

S-derivable:

T-derivable:

U-derivable:

Productions removed:

Productions added:

Remove $A \rightarrow S_1 a S_2$

$A \rightarrow S_1 a S_2$

$a \in \sum$, S_1 and S_2 strings, at least one is not empty

Create

$X_a \rightarrow a$

$A \rightarrow S_1 X_a S_2$

Then fix up $A \rightarrow S_1 X_a S_2$

– why?

Remove $A \rightarrow S_1 X_a S_2$

$A \rightarrow S_1 X_a S_2 S_3$

Put in CNF

- $S \rightarrow ASA \mid aB$
- $A \rightarrow B \mid S$
- $B \rightarrow B \mid \varepsilon$