15. Multiplication

Chapter 10, section 10.3

MULTIPLICATION OF UNSIGNED INTEGERS

Unsigned Integer Multiplication

1011	**Multiplicand (11)**
×1101	**Multiplier (13)**

Computerized Multiplication

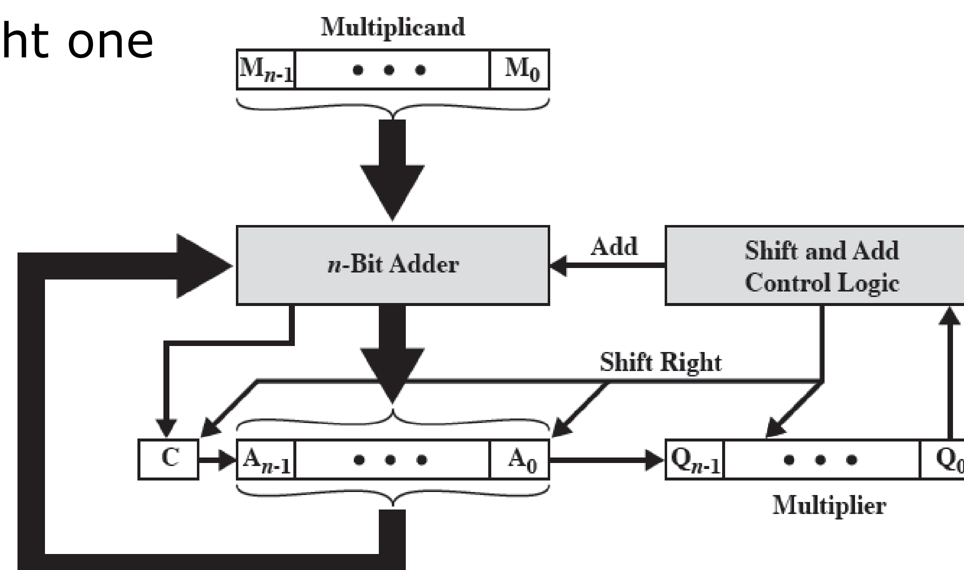
- How can we make multiplication more efficient?

  1. Perform a running addition rather than adding once at the end

  2. We can save time on partial products by shifting

Unsigned Integer Multiplication

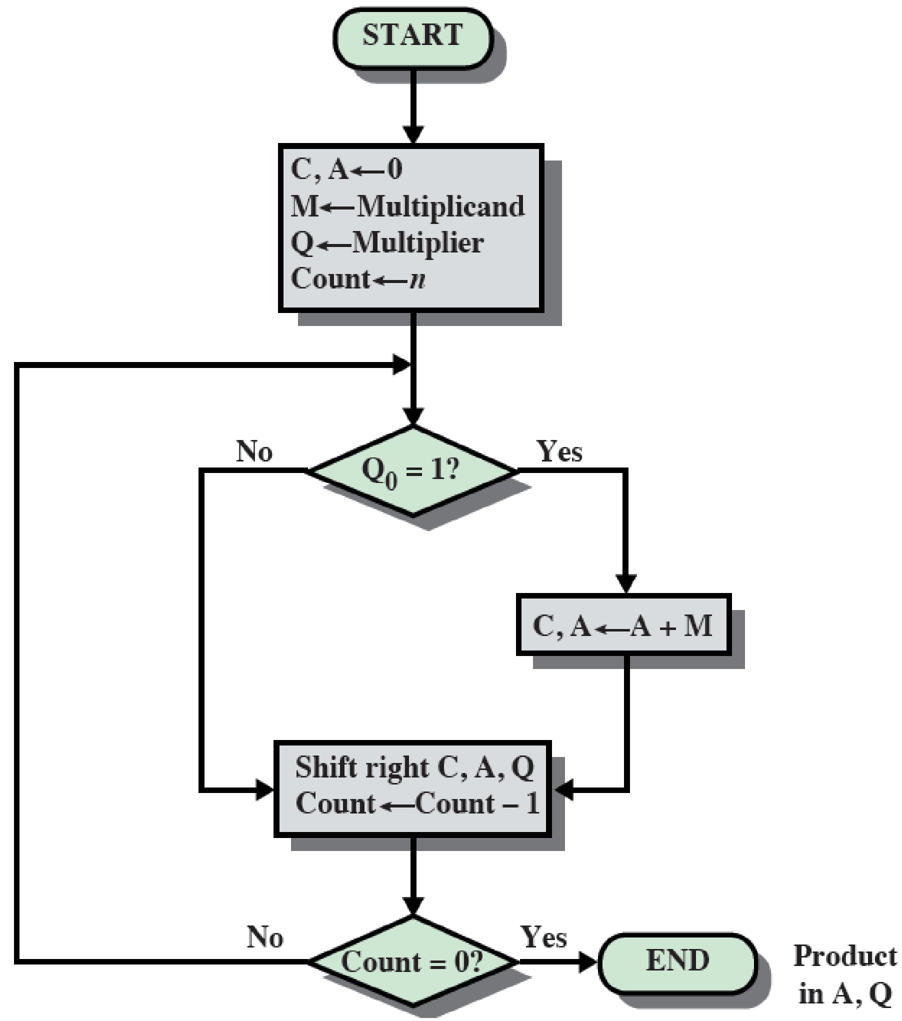
- The multiplier and multiplicand are loaded into Q and M respectively and a third register (A) is needed and initially set to 0.
- Read multiplier bit one at a time
- If Q_0 is 1, then multiplicand is added to A register and the result is stored in A with C used for overflow.
- If Q_0 is 0, then no addition is performed.
- Shift C, all A, and all Q bits right one bit
- Repeat from 1 until each bit in original multiplier is processed



```
  1011
x 1101
------
```

Example

C	A	Q	M	



MULTIPLICATION OF 2'S COMPLEMENT INTEGERS

Unsigned Integer Multiplication

- What would happen if we interpret the following values as 2’s complement values?

```
     1011     Multiplicand (11)
   × 1101     Multiplier (13)
   ------
     1011   ⎫
    0000    ⎬ Partial products
   1011     ⎪
  1011      ⎭
 --------
 10001111     Product (143)
```

2’s Complement Multiplication

- Straightforward multiplication will ***not*** work if either the multiplicand or the multiplier are negative

Unsigned Integer Multiplication

- Here is another way of looking at unsigned integer multiplication:

```
    1011
   ×1101
   -----
    1011
   0000
  1011
 1011
 -------
 10001111
```

1011	
×1101	
00001011	$1011 \times 1 \times 2^0$
00000000	$1011 \times 0 \times 2^1$
00101100	$1011 \times 1 \times 2^2$
01011000	$1011 \times 1 \times 2^3$
10001111	

Another Example

- Multiply the following two numbers showing what is happening in terms of powers of 2:

  0110 X 0110

Negative Multiplicand

- Multiply the following two numbers showing what is happening in terms of powers of 2:

  1011 X 0010

- Is the solution correct? How can we fix it?

Negative Multiplier

- Multiply the following two numbers showing what is happening in terms of powers of 2:

  0101 X 1101

- What is causing the incorrect solution?

2’s Complement Multiplication

- One solution: convert both multiplier and multiplicand to positive numbers, perform the multiplication, and the take the 2’s complement of the result
  - Complicated and expensive

- Another solution: Booth’s Algorithm
  - Developed by Andrew Donald Booth in 1950 in London
  - Used desk calculators that were faster at shifting than adding and created the algorithm to increase their speed

```
  0101
x 1101
------
```

Booth’s Algorithm

A	Q	Q_{-1}	M