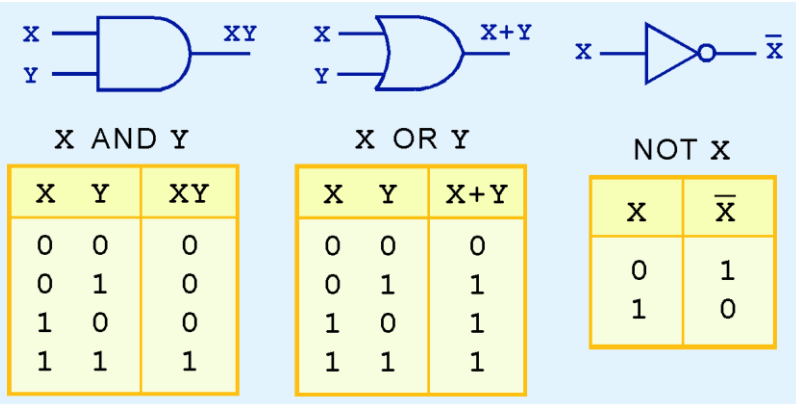
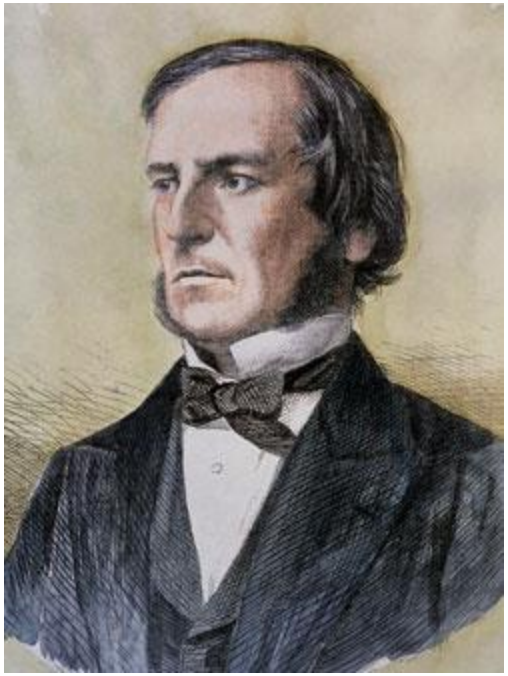


3. Digital Logic

Chapter 11
Boolean Algebra and Gates

Sections 11.1, 11.2

- Reading: pp.365-370
- Good Problems to Work: 11.1 a., 11.3 a.

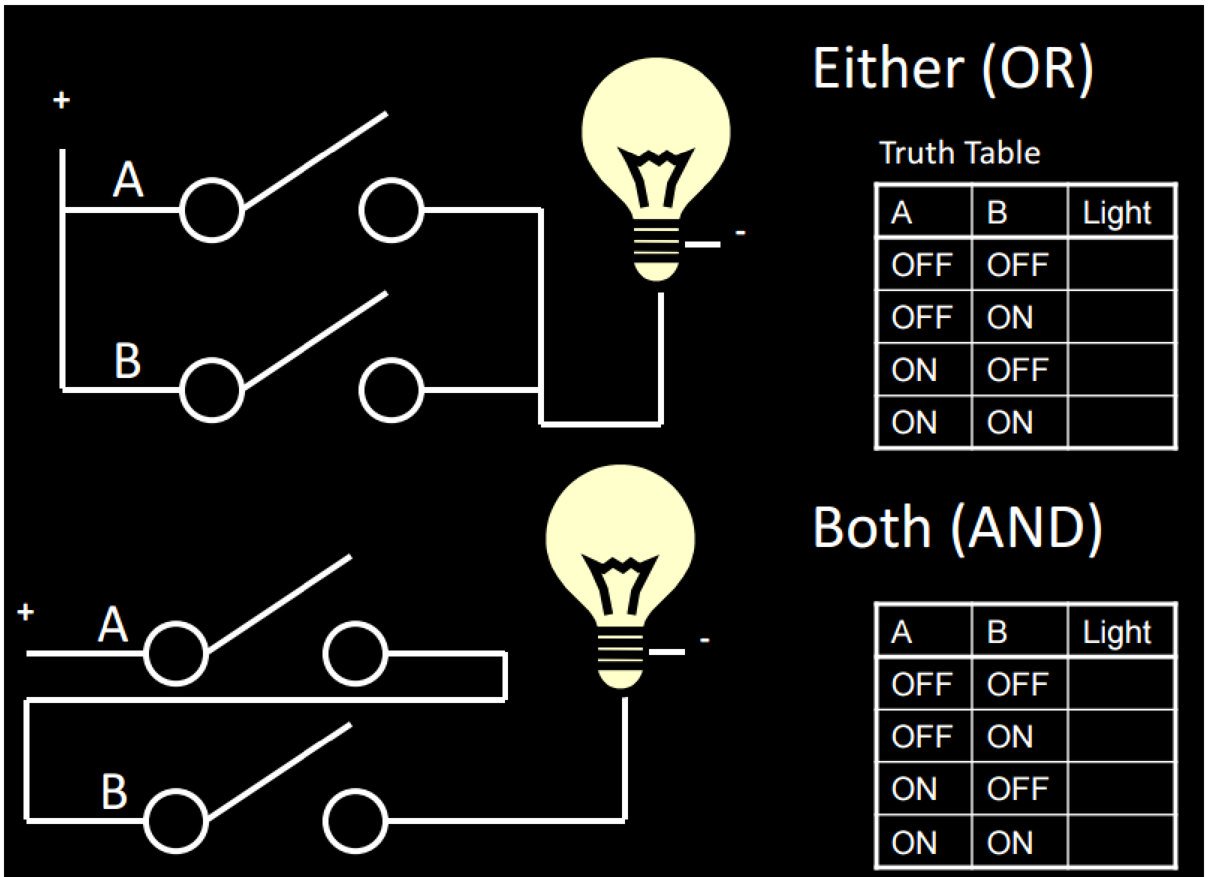


BOOLEAN ALGEBRA

Boolean Algebra

- Boolean algebra is a convenient tool for:
  1. describing the function of digital circuitry
  2. simplifying the implementation of said function
- Boolean algebra uses
  1. logical variables (values are 0 or 1)
  2. operations on logical variables

AND, OR


+
A
B
-
Either (OR)
Truth Table

A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

+
A
B
-
Both (AND)

A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

Boolean Operators of Two Input Variables

P	Q	NOT P ($\overline{P}$)	P AND Q ($P \bullet Q$)	P OR Q ($P + Q$)	P NAND Q ($\overline{P \bullet Q}$)	P NOR Q ($\overline{P + Q}$)	P XOR Q ($P \oplus Q$)
0	0	1	0	0	1	1	0
0	1	1	0	1	1	0	1
1	0	0	0	1	1	0	1
1	1	0	1	1	0	0	0

Truth table of each logical operation

Logical Operators

- AND can be $A * B, A \cdot B, or\ AB$
- OR is $A + B$
- NOT can be $A' or\ \bar{A}$

- Note: In the absence of parentheses, AND takes precedence over OR

Boolean Operators of n Input Variables

Operation	Expression	Output = 1 if
AND	A • B • …	All of the set {A, B, …} are 1.
OR	A + B + …	Any of the set {A, B, …} are 1.
NAND	$\overline{A \bullet B \bullet K}$	Any of the set {A, B, …} are 0.
NOR	$\overline{A + B + K}$	All of the set {A, B, …} are 0.
XOR	A ⊕ B ⊕ …	The set {A, B, …} contains an odd number of ones.

$\overline{A+B} = \overline{A} \bullet \overline{B}$

Identities of Boolean Algebra

Basic Postulates		
$A \bullet B = B \bullet A$	$A + B = B + A$	Commutative Laws
$A \bullet (B + C) = (A \bullet B) + (A \bullet C)$	$A + (B \bullet C) = (A + B) \bullet (A + C)$	Distributive Laws
$1 \bullet A = A$	$0 + A = A$	Identity Elements
$A \bullet \overline{A} = 0$	$A + \overline{A} = 1$	Inverse Elements
Other Identities		
$0 \bullet A = 0$	$1 + A = 1$	
$A \bullet A = A$	$A + A = A$	
$A \bullet (B \bullet C) = (A \bullet B) \bullet C$	$A + (B + C) = (A + B) + C$	Associative Laws
$\overline{A \bullet B} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A} \bullet \overline{B}$	DeMorgan's Theorem

Problem

1. Prove $A + 1 = 1$

2. Prove $A + AB = A$

3. Prove $A + AB = A$ using perfect induction (complete truth table)

Problem

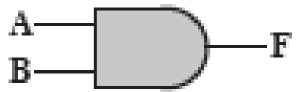
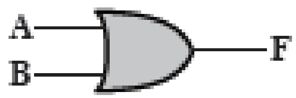
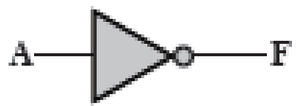
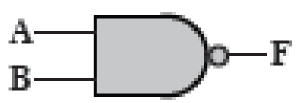
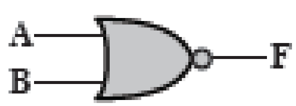
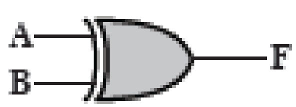
- The complement of any Boolean expression can be found using DeMorgan's Theorem
  1. Find the complement of $X + (YZ)$
  2. Using a truth table, show you really found the complement

GATES

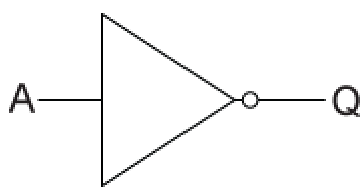
Gates

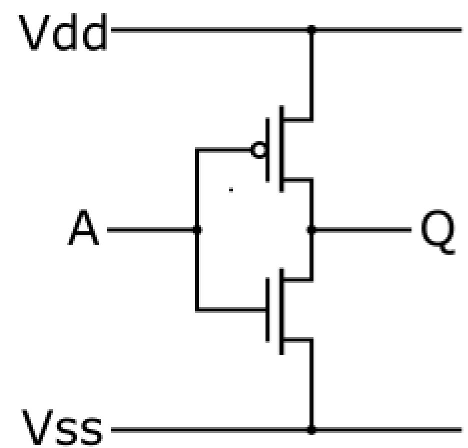
- Gate
  1. an electronic circuit that operates on one or more signals producing an output signal
  2. fundamental building block of ALL digital logic circuits
- Interconnections of these gates allow us to build simple or complicated logical functions

Basic Logic Gates

Name	Graphical Symbol	Algebraic Function	Truth Table
AND		$F = A \cdot B$ or $F = AB$	A B | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = A + B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT		$F = \overline{A}$ or $F = A'$	A | F 0 | 1 1 | 0
NAND		$F = \overline{AB}$	A B | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = \overline{A + B}$	A B | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
XOR		$F = A \oplus B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

Gate as Transistor





Transistors and Gates

- Interested in how to build logic gates from transistors?
  - Watch this: https://www.youtube.com/watch?v=NWhk9qGn8qw

Next Time

- Read section 11.3: pp. 370-380