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# Medians and Order Statistics

## Chapter 9

# Order Statistics

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- Select the  $i_{\text{th}}$  smallest of  $n$  elements (the element with rank  $i$ ).
  - Minimum:  $i =$
  - Maximum:  $i =$
  - Median:  $i =$
- What is a naive algorithm for this problem?
- What is its worst-case running time?



# Max and Min

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- How many comparisons are needed to find Max and Min independently?
- Can we do better?

# Simultaneous Max and Min

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- At most  $3n/2$  comparisons are needed

# Analysis

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- Total number of comparisons when:
  - $n$  is odd:
  
  
  
  
  
  
  
  
  
  
  - $n$  is even:

# Example

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- $n = 5, A = \langle 2, 7, 1, 3, 4 \rangle$

# Example

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- $n = 6, A = \langle 2, 5, 3, 7, 1, 4 \rangle$



# Order Statistics

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- RANDOMIZED-SELECT( $A, p, r, i$ )

# Example

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- A:  $\langle 6, 10, 13, 5, 8, 3, 2, 11 \rangle$

# Selection in worst-case Linear Time

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- The worst-case for RANDOMIZED-SELECT is  $n^2$
- Can we do better?

# SELECT

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One iteration on the list {0,1,2,3,...99}

	12	15	11	2	9	5	0	7	3	21	44	40	1	18	20	32	19	35	37	39
	13	16	14	8	10	26	6	33	4	27	49	46	52	25	51	34	43	56	72	79
<b>Medians</b>	17	23	24	28	29	30	31	36	42	47	50	55	58	60	63	65	66	67	81	83
	22	45	38	53	61	41	62	82	54	48	59	57	71	78	64	80	70	76	85	87
	96	95	94	86	89	69	68	97	73	92	74	88	99	84	75	90	77	93	98	91

[http://en.wikipedia.org/wiki/Selection\\_algorithm](http://en.wikipedia.org/wiki/Selection_algorithm)