Integer Arithmetic

Reading: pp. 296-302

Addition

In general, we know the following is true:

P1: Perform the following addition and interpret the result in: (a) modulo 2ⁿ and (b) two's complement notation.

010010101	10101010
+000111001	+10101010

We must become familiar with the use of the carry. There are a couple of different carries that we must concern ourselves with. The carry-in and carry-out.

P2: Add the following two binary values discussing what is meant by carry-in and carryout.

00001111 +01010101

Note: The carry-out of the MSB is the value that the external carry flag found in the flags register takes on.

Q1: In the previous example, what would be the value of the external carry? Why?

Half-Adders and Adders

Subtraction

Subtraction works a little differently than one would think. In particular, subtraction is performed by taking the two's complement of the subtrahend and adding this value to the minuend. Let's look at the following example for some clarification.

Perform the following subtraction:

00110011 (Minuend) -00001111 (Subtahend)

Q2: Before performing the subtraction, identify the two numbers being subtracted. Assume the numbers are represented in modulo 2^n .

P3: Now perform the subtraction.

Q3: Interpret the result. Is it what you would expect it to be?

P4: Switch the minuend and the subtrahend from the above problems and then perform the subtraction. Interpret the result if the numbers are: (a) modulo 2ⁿ and (b) two's complement numbers.

Arithmetic Overflow

Remember that the range of values that can be represented using 8-bits for modulo 2^n numbers is 0 to 255 and for two's complement is -128 to 127. The microprocessor will perform the addition or subtraction of two numbers, but the question is how do we know if the result is correct. That is, if we add two 8-bit numbers and the result is larger than the representation allows, how do we know this happened. The answer lies with two flags: (a) the carry flag and (b) the overflow flag.

First we will define overflow as a condition such that an arithmetic operation produces a result outside the range of the number system being used.

P5: Perform the operations below and interpret the result in: (a) modulo 2ⁿ and (b) two's complement notation.

11111111 01111111 +00000001 +00000001 _____

Q4: Were there any examples of overflow? Identify each case and briefly explain why.

Test each example using debug in Windows XP.

🔤 C:\WINDOWS\system32\cmd.exe - debug	
C:\DOCUME~1\ryandj>debug	
HX=0000 BX=0000 CX=0000 DX=0000 SP=FFEE BF=0000 SI=0000 DI=0000 DS=0BFC ES=0BFC SS=0BFC CS=0BFC IP=0100 NV UP EI PL NZ NA PO NC 0BFC:0100 B0AA MOV AL,AA -a	
0BFC:0100 mov al,ff 0BFC:0102 mov bl,1 0BFC:0104 add al,bl 0BFC:0106 -g =100 106	
AX=0000 BX=0001 CX=0000 DX=0000 SP=FFEE BP=0000 SI=0000 DI=0000 DS=0BFC ES=0BFC SS=0BFC CS=0BFC IP=0106 NV UP EI PL ZR AC PE CY 0BFC:0106 893E7891 MOV [9178],DI DS:917	8=0000

🔤 C:\WINDOWS\system32\cmd.exe - debug	
C:\DOCUME~1\ryandj>debug -r	
AX=0000 BX=0000 CX=0000 DX=0000 SP=FFEE BP=0000 SI= DS=0BFC ES=0BFC SS=0BFC CS=0BFC IP=0100 NV UP EI PI 0BFC:0100 B000 MOV AL,00 -a	0000 DI=0000 NZ NA PO NC
0BFC:0100 mov al,7f 0BFC:0102 mov bl,1 0BFC:0104 add al,bl 0BFC:0106 -g =100 106	
AX=0080 BX=0001 CX=0000 DX=0000 SP=FFEE BP=0000 SI= DS=0BFC ES=0BFC SS=0BFC CS=0BFC IP=0106 OV UP EI NC 0BFC:0106 893E7891 MOV [9178],DI	0000 DI=0000 NZ AC PO NC DS:9178=0000

Let's perform the following additions and determine where any overflows occurred.

1001	1100
+0101	+0100
0011	1100
+0100	+1111
0101	1001
+0100	+1010

Let's perform the following subtractions and determine where any overflows occurred. Represent the numbers in 2's complement.

2 – 7	5 - 2
(-5) - 2	5 - (-2)
7 – (–7)	(-6) - 4

For each of the following, perform the additions, convert the binary numbers to decimal where the binary numbers are represented in two's complement, and indicate where a carry-out or overflow occurs.

11011001	11101101	00101100
+01011100	+11111001	+00101101
01101000	10110101	10011001
+00101101	+00111011	+10111011
00001010	01111111	1111111
+1111101	+00000001	+00000001