Chapter 9 – Computer Arithmetic

Reading: Section 9.1 on pp. 290-296

Computer Representation of Data

Groups of two-state devices are used to represent data in a computer. In general, we say the states are either: high/low, on/off, 1/0, ...

P#1: How many combinations can be represented by a group of three of these two-state devices?

P#2: Give a general formula for the number of combinations of a group of N of these two-state devices.

We will assume that the memory word size is 8-bits for the purposes of discussion. You should be able to apply the concepts to memory cell sizes of n-bits.

Unsigned Numbers (Modulo 2^n)

Q#1: How many different unsigned numbers can be represented in 8-bits?

P#1: Let's write out the first 3 unsigned integers and the last 2 under the following column headings.

Bit Pattern	Modulo(2^8) Integer

It is important to note that the Intel processors use the modulo 2^n number system to represent unsigned integers. It is also important to know that in C++ when an integer is declared as say: unsigned int x; that x is represented in modulo 2^n representation.

Q#2: How many bytes are allocated to an unsigned int in C++?

Signed Numbers

Signed Magnitude

Signed magnitude numbers provide for both positive and negative integers. The leftmost bit (MSB - most significant bit) is the sign bit. A zero signifies a positive integer and a one signifies a negative integer. For example, 00001111 represents a +15 and 10001111 represents a -15.

Q#1: What is the range of integers that can be represented in 8-bits?

Q#2: What is the general formula for this range using N-bits?

Q#3: How many representations of zero exist?

Q#4: What is the representation of 127.

Q#5: If we add one to the value in Q#4, what do we end up with?

P#1: Fill in the table (first 2 & last 2) below:

Bit Pattern	Signed Magnitude

One's Complement

One's complement numbers provide for both positive and negative integers. The MSB is the sign bit. A zero signifies a positive integer and a one signifies a negative integer. For example, 00001111 represents a +15 and but 10001111 does not represent a -15. How do we find out what 10001111 represents? The answer is that we invert the bits, add up the positional values of each bit, and put a negative sign in front of the number.

Q#1: What is the range of integers that can be represented in 8-bits?

Q#2: What is the general formula for this range using N-bits?

Q#3: How many representations of zero exist?

Q#4: What is the representation of -127.

Q#5: If we add one to the 127, what do we end up with?

P#1: Fill in the table (first 2 & last 2) below:

Bit Pattern	One's Complement

Two's Complement

Two's complement numbers provide for both positive and negative integers. The MSB is the sign bit. A zero signifies a positive integer and a one signifies a negative integer. For example, 00001111 represents a +15 and but 10001111 does not represent a -15. In order to find out what this bit pattern represents, take the one's complement and add 1. Sum up the bits and convert to decimal. Finally, place a negative sign in front of the number.

So what decimal number does the two's complement number 10001111 represent?

Q#1: What is the range of integers that can be represented in 8-bits?

Q#2: What is the general formula for this range using N-bits?

Q#3: How many representations of zero exist?

Q#4: What is the representation of -127.

Q#5: If we add one to the value 127, what do we end up with?

P#1: Fill in the table (first 2 & last 2) below:

Bit Pattern	Two's Complement
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General Problems

1. What is the smallest integer that can be represented in a 12-bit two's complement format?
2. Consider the number $(9A)_{16}$. Is it positive or negative, if it represents an 8-bit two's complement integer?
3. What is the 8-bit two's complement format for $(-27)_{10}$
4. If 01101010 is an 8-bit two's complement integer, what is it's decimal equivalent?
5. What base 10 value does 1011 represent? Give the answer for the following representation systems. Assume each system uses 4 bits.
    a. UB (unsigned binary)
    b. 1C (one's complement)
    c. 2C (two's complement)
    d. SM (signed magnitude)
6. Translate -4_{10} to the following representations. Assume 4 bits. Answers using fewer or more than 4 bits will not be considered correct. If it's not possible to represent -4 in a given representation system, write NR (not representable).
    a. UB (unsigned binary)
    b. 1C (one's complement)
    c. 2C (two's complement)
    d. SM (signed magnitude)
7. Using 8 bit binary numbers, represent the decimal number -18 in :
    a. two's complement representation
    b. signed magnitude representation
8. Given the following choice of representation for signed numbers:
    a. 1's complement
    b. 2's complement
    c. Sign and magnitude
    d. None of the above
9. If numbers are 8 bits wide, for each of the bit patterns shown below, write the letter (a,b,c,d) corresponding to the representation in which each is interpreted as -23(base 10)
    a. 11101001____
    b. 10010111____
    c. 11101000____
    d. 11101010____
10. What is the value in decimal of the most negative 8-bit 2's complement integer?
11. What is the value in decimal of the most positive 8-bit unsigned integer?