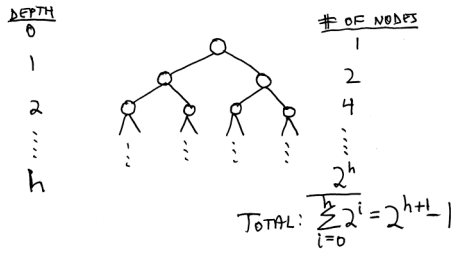
Heapsort

Chapter 6

Review of Binary Trees

- What is a binary tree?
- What is the depth of the node?
- What is the height of a node?
- What is the height of the tree?
- What is a complete binary tree?

Facts about Complete Binary Trees



Heaps

- A ***heap*** is an "*almost*" complete binary tree
- Extra nodes go from left to right at the lowest level
- Where the value at each node is ≥ the values at its children (if any)
- This is called the ***heap property*** for max-heaps

Example

Storing Heaps

- As arrays!
- Root of tree is:
- Parent of A[i] is:
- Left child of A[i] is:
- Right child of A[i] is:

Example

- n = 13

92 85 73 81 44 59 64 13 23 36 32 18 54

Functions on Heaps

- MAX-HEAPIFY
- BUILD-MAX-HEAP
- HEAPSORT
- MAX-HEA-INSERT
- HEAP-EXTRACT-MAX
- HEAP-INCREASE-KEY
- HEAP-MAXIMUM

MAX-HEAPIFY

Example

- 15 6 4 8 5 3 1 2 7 i = 2

BUILD-MAX-HEAP

Example

- 4 3 7 13 1 20 12 16 2 18

HEAPSORT

Example

- 20 18 12 16 3 7 4 13 2 1

Priority Queues

- Priority Queues are an example of an application of heaps.
- A priority queue is a data structure for maintaining a set of elements, each with an associated key.

Priority Queues

- Max-priority queue supports dynamic set operations:
  - INSERT(S, x): inserts element x into set S.
  - MAXIMUM(S): returns element of S with largest key.
  - EXTRACT-MAX(S): removes and returns element S with largest key.
  - INCREASE-KEY(S, x, k): increases value of element x's key to k. Assume k >= x's current key value.

HEAP-MAXIMUM(A)

HEAP-EXTRACT-MAX

- Given the array A:
  - Make sure heap is not empty.
  - Make a copy of the maximum element.
  - Make the last node in the tree the new root.
  - Re-heapify the heap, with one fewer node.
  - Return the copy of the maximum element.

Example

- 15 6 4 8 5 3 1 2 7

HEAP-INCREASE-KEY

- Given set S, element x, and new key value k:
  - Make sure >= x's current key.
  - Update x's key value to k.
  - Traverse the tree upward comparing x to its parent and swapping keys if necessary, until x's key is smaller than its parent's key.

Example

- Increase key of node 6 in previous example to 20

MAX-HEAP-INSERT

- Given a key k to insert into the heap:
  - Insert a new node in the very last position in the tree with the key -infinity.
  - Increase the -infinity key to k using the HEAP-INCREASE-KEY procedure.

Example

- Insert 12 into the above heap.