Math122 College Algebra

Professor Douglas J. Ryan

4.1
Quadratic Functions & Models

- A quadratic function is a polynomial function of degree 2.

- The form of a quadratic function is
  $$f(x) = ax^2 + bx + c \ (a \neq 0)$$

- Standard form of a quadratic function is
  $$f(x) = a(x - h)^2 + k$$

Graph of a Quadratic

- The graph of a quadratic f is a parabola with vertex (h, k)

- If $a > 0$ the parabola opens upward

- If $a < 0$ the parabola opens downward

Problem

- Let $f(x) = 2x^2 - 8x + 12$

  1. Express f in standard form

  2. Sketch the graph of f

Maximum & Minimum Values Of Quadratic Functions

- If $f(x) = a(x - h)^2 + k$, the maximum or minimum value of f occurs at $x = h$

- If $a > 0$, the minimum value of f is $f(h) = k$, that is, the minimum value of f is at (h, k)

- If $a < 0$, the maximum value of f is $f(h) = k$, that is, the maximum value of f is at (h, k)

- Draw two graphs that generalizes each of these facts. That is, what is actually being said visually.

Problem

- Consider the quadratic function $f(x) = 5x^2 - 30x + 49$

  1. Express f in standard form
  2. Find the minimum
  3. Find the x-intercepts
  4. Find the y-intercepts
  5. Sketch the graph

General Maximum & Mininimum Values of Quadratics

- The minimum or maximum value of a quadratic $f(x) = ax^2 + bx + c$ $(a \neq 0)$ occurs at $x = -\dfrac{b}{2a}$

- If $a > 0$, the minimum is $f\left(-\dfrac{b}{2a}\right)$

- If $a < 0$, the maximum is $f\left(-\dfrac{b}{2a}\right)$

Problems

- Find the maximum or minimum value of
  1. $f(x) = x^2 + 6x$
  2. $g(x) = -2x^2 + 4x - 6$

- The revenue generated by a manufacturer selling x units of a product is $P(x) = -0.4x^2 + 80x$. How many units should be sold to obtain the maximum revenue?