Math122 College Algebra

Professor Douglas J. Ryan

1.7
Absolute Value Equations/Inequalities

- Remember $|a| = \begin{cases} a \ if \ a \geq 0 \\ -a \ if \ a < 0 \end{cases}$

- $|a|$ is the distance from a to the origin on the real number line

- $|x - a|$ is the distance between x and a on the real number line

Absolute Value Equations

- The property $|x| = C$ is equivalent to $x = \pm C$

- That is, to solve an absolute value equation, two separate equations must be solved

- Example, $|x| = 2$ is equivalent to $x = 2$ or $x = -2$

Sample Problem

- Solve the equation $|2y + 6| = 4$
- Answer

$$2y + 6 = 4 \qquad \text{or} \qquad 2y + 6 = -4$$
$$2y = -2 \qquad\qquad\qquad 2y = -10$$
$$y = -1 \qquad\qquad\qquad y = -5$$

- Check

$$|2 \cdot -1 + 6| = |-2 + 6| = |4| = 4$$
$$|2 \cdot -5 + 6| = |-10 + 6| = |-4| = 4$$

Problem

- Solve $4|y - 2| - 2 = 8$

- Check your solution

Absolute Value Inequalities

- Properties of Absolute Value Inequalities

1. $|x| < c \iff -c < x < c \; (e.g. \, |x| < 2)$

2. $|x| \leq c \iff -c \leq x \leq c$

3. $|x| > c \iff x > c \text{ or } x < -c \; (e.g. \, |x| > 2)$

4. $|x| \geq c \iff x \geq c \text{ or } x \leq -c$

Sample Problem

- Solve the inequality $|y - 2| < 4$
- Answer

$-4 < y - 2 < 4$
$-2 < y < 6$
$(-2, 6)$

Problem

- Solve each of the following problems and give your answer in interval notation.

1. $|a - 3| < 0.1$

2. $|2y + 3| > 9$