Math122 College Algebra

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1.6
Inequalities

- Some problems lead to inequalities instead of equations.
- Equations have a fixed number of solutions
- Inequalities tend to have infinitely many solutions
  - ➢Equation $4x + 7 = 15$ solution $x = 2$
  - ➢Inequality $4x + 7 \leq 15$solution $x \leq 2$

Rules for Inequalities

1. $A \leq B \Leftrightarrow A + C \leq B + C$
2. $A \leq B \Leftrightarrow A - C \leq B - C$
3. $If\ C > 0, then\ A \leq B \Leftrightarrow CA \leq CB$
4. $If\ C < 0, then\ A \leq B \Leftrightarrow CA \geq CB$
5. $If\ A > 0\ and\ B > 0$

$$then\ A \leq B \Leftrightarrow \frac{1}{A} \geq \frac{1}{B}$$

6. $If\ A \leq B\ and\ C \leq D, then\ A + C \leq B + D$

Linear Inequality

- Solve the inequality $-3x + 4 > 11$
- Answer

$$-3x + 4 > 11$$
$$(-3x + 4) - 4 > 11 - 4$$
$$-3x > 7$$
$$\left(-\frac{1}{3}\right)(-3x) < \left(-\frac{1}{3}\right)(7)$$
$$x < -\frac{7}{3} \; which \; is \; \left(-\infty, -\frac{7}{3}\right)$$

Problem

- Solve each of the following inequalities:

1. $4x - 3 < 2x + 5$

2. $-6 < 2y - 4 < 2$

Solving Nonlinear Inequalities

1. Factor

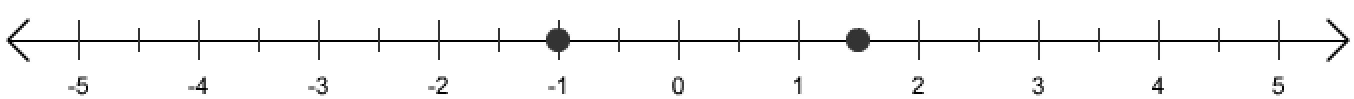
2. If a product (or quotient) has an even number of negative factors, the value is positive

3. If a product (or quotient) has an odd number of negative factors, the value is negative

Sample Problem

Solve $2x^2 - x < 3$

1. Factoring yields $2x^2 - x - 3 < 0 \quad \Leftrightarrow$
$$(x+1)(2x-3) < 0$$
2. $2x^2 - x - 3 = 0 \; has \; solutions$
$$x = -1 \; and \; x = \frac{3}{2}$$
3. The real axis can be divided into three parts as follows: $(-\infty, -1), \left(-1, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$



Sample Problem

- Next we determine the sign of each factor on each of the intervals

Interval	$(-\infty, -1)$	$(-1, 3/2)$	$(3/2, \infty)$
Sign of $x + 1$	$-$		
Sign of $2x - 3$	$-$		
Sign of $(x + 1)(2x - 3)$	$+$		

- Fill in the rest of the table

Sample Problem

- A different way to represent the information from the previous slide

Interval	$(-\infty, -1)$	$(-1, 3/2)$	$(3/2, \infty)$
k	-2	0	2
Value of $2x^2 - x - 3$ at k	$+7$		
Sign of $2x^2 - x - 3$ at k	$+$		

- Fill in the rest of the table

Sample Problem

- Using either table, we see that $\left(-1, \frac{3}{2}\right)$ are solutions to $2x^2 - x < 3$

- Note: Since the inequality is strictly less than, we do not include -1 or $\frac{3}{2}$ in the solution set.

Problem

- Your turn … Solve $y^2 \geq 7y - 10$. Represent your solutions: (a) using interval notation and (b) graphically

Interval			
k			

Problem

- Solve $\frac{x+5}{x+3} \geq 0$. Represent your solutions: (a) using interval notation and (b) graphically

Interval			
k			

Problem

- A package of food states that the food should be stored at a temperature of 5 degrees Celsius and 20 degrees Celsius inclusive. The relationship between Celsius and Fahrenheit is $C = \frac{5}{9}(F - 32)$. What range of temperatures does this correspond to in Fahrenheit?