



Math122 College Algebra

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P.8

Rational Expressions

- Fractional expression is the quotient of two algebraic expressions $\frac{2x}{x+1}$ $\frac{\sqrt{y}}{y-1}$
- Rational expression is the quotient of two polynomials $\frac{2x}{x+1}$ $\frac{y}{y^2+1}$
- Is $\frac{\sqrt{y}}{y-1}$ a rational expression? Why or why not?

Domain of Algebraic Expression

- The domain of an algebraic expression is the set of real numbers the variable can have
- What is the domain for

1. $\frac{1}{y}$

2. \sqrt{y}

3. $\frac{1}{\sqrt{a}}$

4. $\frac{2b+3}{b+3}$

Problem

- Find the domain for each of the following expressions and give your answer in interval notation

1. $y^2 - 1$

2. $\frac{a}{a^2 - 1}$

3. $\frac{\sqrt{b}}{b+1}$

Simplifying Rational Expressions

- Simplifying rational expressions uses the following property of fractions:
- $\frac{AC}{BC} = \frac{A}{B}$
- We find common factors in the numerator and denominator and then cancel the common factors

Multiplying and Dividing Rational Expressions

- Remember the following properties of fractions:

$$1. \frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD} \text{ where } B, D \neq 0$$

$$2. \frac{A}{B} \div \frac{C}{D} = \frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C} \text{ where } B, D, C \neq 0$$

Problem

- Perform the operation and simplify:

1. $2^{-2} \cdot 3^{-2}$

2. $\frac{x^2-1}{x^2-x-6} \cdot \frac{x^2-4x+3}{x^2+5x+4}$

3. $2^{-2} \div 3^{-2}$

4. $\frac{x^2-x-6}{x^2+2x-15} \div \frac{x-2}{x+5}$

Adding and Subtracting Rational Expressions

- Remember the following properties of fractions:

$$1. \frac{A}{C} + \frac{B}{C} = \frac{A+B}{C} \text{ where } C \neq 0$$

$$2. \frac{A}{C} - \frac{B}{C} = \frac{A-B}{C} \text{ where } C \neq 0$$

- $\text{T/F } \frac{A}{B+C} = \frac{A}{B} + \frac{A}{C}$

Problem

- Perform the operation and simplify:

1. $2^{-2} + 3^{-2}$

2. $\frac{y}{y+3} + \frac{2}{y-2}$

3. $\frac{a}{a^2-4} + \frac{2a}{a^2-5a+6}$

Problem

- Perform the operation and simplify

1. $\frac{3x+1}{2x-5} - \frac{x-3}{2x-5}$

2. $\frac{1}{y+3} - \frac{2}{y^2-9}$

Compound Fractions

- A compound fraction is a fractional expression where the numerator and/or denominator is a fractional expression

- Simplify $\frac{\frac{2}{x+1}}{\frac{5}{x^2-1}}$

- Simplify $\frac{1+\frac{1}{y}}{2-\frac{1}{y}}$

Compound Fractions

- Simplify by multiplying the numerator and denominator by the LCD

1. $\frac{\frac{x}{y} + 1}{1 - \frac{y}{x}}$

Rationalizing the Denominator

- Consider a fractional expression with a denominator form $A + B\sqrt{C}$
 - The conjugate radical is $A - B\sqrt{C}$
 - $(A + B\sqrt{C})(A - B\sqrt{C}) =$
- Rationalize the denominator $\frac{1}{1-\sqrt{2}}$

Rationalizing the Numerator

- Rationalize the numerator and simplify:

1. $\frac{\sqrt{4+h}-2}{h}$

2. $\frac{2+\sqrt{8}}{4}$ (tricky ... simplify)