Math122 College Algebra

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P.5
Rational Exponents and Radicals

- We now what 2^3 means
- We now need to discuss such expressions as $8^{\frac{2}{3}}$
- $\sqrt{a} = b$ means $b^2 = a$ and $b \geq 0$

Define n^{th} Root

- Define the n^{th} root as follows
  - Let n be any positive integer, the principal n^{th} root of a is $\sqrt[n]{a} = b$ means $b^n = a$

Note1: if n is even, then we must have $a \geq 0$ and $b \geq 0$

Note2: The principal n^{th} root has the same sign as the original number

Problem

- Evaluate each of the following
  1. $\sqrt[4]{16}$
  2. $\sqrt[3]{-27}$
  3. $\sqrt[2]{(-4)^2}$
- T/F $\sqrt{a^2} = a$ for all a

Properties of n^{th} Roots

1. $\sqrt[n]{ab} =$

2. $\sqrt[n]{\dfrac{a}{b}} =$

3. $\sqrt[m]{\sqrt[n]{a}} =$

Properties of n^{th} Roots

1. $\sqrt[n]{a^n}=$ if n is odd

2. $\sqrt[n]{a^n}=$ if n is even

Problem

- Simplify each of the following:
  1. $\sqrt[4]{x^6}$
  2. $\sqrt[4]{32a^8b^4}$
  3. $\sqrt{32} + \sqrt{200}$

Rational Exponents

- An example of a rational exponent is $a^{\frac{2}{3}}$
- Definition of Rational Exponents
  - For any reduced rational exponent m/n where m and n are integers and $n > 0$,

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$
$$OR$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

  - Note: If n is even, then a requirement is $a \geq 0$

Problem

- Evaluate each of the following:
  1. $8^{\frac{1}{3}}$
  2. $8^{\frac{2}{3}}$
  3. $8^{\frac{-1}{3}}$

Problem

- Simplify each of the following:

    1. $a^{\frac{1}{2}}a^{\frac{1}{3}}$

    2. $\dfrac{a^{\frac{1}{2}}a}{a^{\frac{3}{4}}}$

    3. $(2a^3b^4)^{\frac{3}{2}}$

Problem

- Simplify the following radicals and write your result as a rational exponent
  1. $\left(\sqrt{4x}\right)\left(3\sqrt[3]{x}\right)$
  2. $\sqrt{x\sqrt{x}}$

Rationalizing the Denominator

- Rationalizing the denominator is the process of eliminating all radicals in the denominator
- If the denominator is of the form $\sqrt{a}$ then simply multiply numerator and denominator by $\sqrt{a}$
- $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot 1 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Rationalizing the Denominator

- If the denominator is of the form $\sqrt[n]{a^m}$ and $m < n$ then multiply the numerator and denominator by $\sqrt[n]{a^{n-m}}$

- What is $\sqrt[n]{a^m}\sqrt[n]{a^{n-m}}$

Problem

- Rationalize the denominator for

1. $\frac{3}{\sqrt{5}}$

2. $\frac{2}{\sqrt[3]{3}}$

3. $\frac{1}{\sqrt[3]{3x^2}}$