Math122 College Algebra
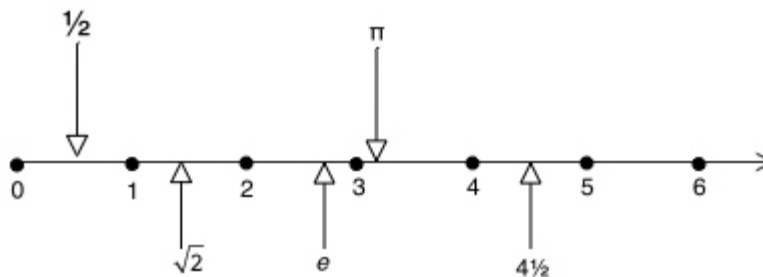
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P.3
The Real Number Line and Order

- Coordinate line, or real number line, or real line

- The real numbers are ordered (e.g. $\sqrt{2} < 2$)
- Graph $x < -5$
- Graph $x \geq 3$

Sets and Intervals

- A set is a collection of elements

- $a \in S$ means a is an element of S

- $a \notin S$ means a is not an element of S

- Write the set A of all positive integers less than 5

  1. $A = \{1,2,3,4\}$

  2. Using set-builder notation
     $A = \{x | x \text{ is an integer and } 0 < x < 5\}$

More Sets

- If S and T are sets
  1. $S \cup T$ is set union (all elements in S or T (or in both)

  2. $S \cap T$ is set intersection (all elements that are in both S and T

  3. $\emptyset$ is the empty set (contains no elements)

Problem

- If S={1,2,3,4,5}, T={4,5,6,7}, and V={6,7,8,9}, find each of the following:

  1. $S \cup T$

  2. $S \cap T$

  3. $S \cap V$

Interval Notation

- Open interval $(a, b) = \{x | a < x < b\}$
- Closed interval $[a, b] = \{x | a \leq x \leq b\}$
- Other interval notation

$[a, b)$ $\qquad$ $(a, b]$

(a, ∞) $\qquad$ $[a, \infty)$

$(-\infty, b)$ $\qquad$ $(-\infty, b)$

$(-\infty, +\infty)$

Problem

- Express each of the following intervals using set-builder notation and then graph the interval

  1. $[1.5, 4)$

  2. $(-1, +\infty)$

Problem

- Given (1,3) ∩ [2,7]

    1. Express using set-builder notation

    2. Graph the set intersection

Absolute Value and Distance

- The absolute value of the number a, denoted by $|a|$ is
  - ➤ the distance from a to 0 on the real number line
  - ➤ always positive or 0
  - ➤ $|a| \geq 0$ for every number a

- $|a| = \begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a < 0 \end{cases}$

Problem

- Remember the definition of $|a|$ from the previous slide
- For each expression below, state what a is and then evaluate the expression

  1. $|3|$

  2. $|-3|$

  3. $|0|$

Problem

- For each expression below, state what a is and then evaluate the expression

  1. $|\sqrt{2}\text{-}1|$

  2. $|3 - \pi|$

Properties of Absolute Value

1. $|a| \geq 0$

2. $|a| = |-a|$

3. $|ab| = |a||b|$

4. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$

Distance Between Points

- The distance between two real numbers a and b on the real line is $d(a, b) = |b - a|$

- Is $|b - a| = |a - b|$?

- If so, why? If not, why not?

Problem

- Evaluate $\left|3 - \left|-1\right|\right|$

- Find $d(-2,5)$

- Find $d\left(\frac{1}{4}, -\frac{1}{12}\right)$