Math122 College Algebra

Professor Douglas J. Ryan

P.2
Real Numbers and Their Properties

- Define each of the following
  - natural numbers
  - integers
  - rational numbers
  - irrational numbers

Real Numbers

- The natural numbers $\mathbb{N}$, integers $\mathbb{Z}$, rational numbers $\mathbb{Q}$, and irrational numbers make up the set of all real numbers $\mathbb{R}$
- The word number without qualifications means real
- Every real number has a decimal representation

Decimal Representations

- All rational numbers have a repeating decimal
  - *a.* $\frac{1}{2} = 0.5\overline{0}$
  - *b.* $\frac{1}{3} = 0.\overline{3}$

- All irrational numbers do not have any repeating decimals
  - *a.* $\sqrt{2} = 1.41421356 \ldots$
  - *b.* $\pi = 3.14159 \ldots$

Operations on Real Numbers

- Evaluating arithmetic expressions
  1. Perform operations inside parentheses
     a. innermost to outermost
     b. treat the numerator and denominator of a fraction as being with parentheses
  2. Perform multiplications and divisions from left to right
  3. Perform additions and subtractions from left to right

Problem

- Evaluate $-2 + [4 \cdot 7 - 5\left(9 - \frac{8}{2}\right)$

Properties of Real Numbers

- Commutative
  - Addition:
  - Multiplication:
- Associative
  - Addition
  - Multiplication:
- Distributive

Properties of Negative Numbers

1. $(-1)a = -a$
2. $-(-a) = a$
3. $(-a)b = a(-b) = -(ab)$
4. $(-a)(-b) = ab$
5. $-(a + b) = -a - b$
6. $-(a - b) = -a + b = b - a$

True / False $-(x + y - z) = x - y + z$

Properties of Fractions

1. $\frac{a}{b} \cdot \frac{c}{d} =$

2. $\frac{a}{b} \div \frac{c}{d} =$

3. $\frac{a}{c} + \frac{b}{c} =$

Properties of Fractions

4. $\frac{a}{b} + \frac{c}{d} =$

5. $\frac{ac}{bc} =$

6. if $\frac{a}{b} = \frac{c}{d}$ then

Least Common Denominator (LCD)

- Evaluate $\frac{5}{36} + \frac{7}{120}$
- Answer #1: Use Property 4 from previous slide
- Answer #2:
  - Factor each denominator into prime factors
    $$36 = 2^2 \cdot 3^2; 120 = 2^3 \cdot 3^1 \cdot 5^1$$
  - Form LCD taking highest power of each factor
    $$2^3 \cdot 3^2 \cdot 5^1 = 360$$
  - Now what?

Other Terminology

- Addition & Subtraction
  a) 0 is the additive identity as $a + 0 = a$
  b) every $\mathbb{R}$ a has a negative $-a$ such that
  $$a + (-a) = 0$$
- Multiplication & Division
  a) 1 is the multiplicative identity as $a \cdot 1 = a$
  b) every nonzero $\mathbb{R}$ a has an inverse $\frac{1}{a}$ such that
  $$a \cdot \left(\frac{1}{a}\right) = 1$$

Other Terminology

- Consider $\frac{a}{b}$
  a) is the quotient of a and b
  b) is the fraction a over b
  c) a is the numerator
  d) b is the denominator (or divisor)

Problem

- Evaluate without a calculator

1. $3 + \frac{7}{8} - \frac{5}{6}$

2. $0.30\left(\frac{4}{3} + \frac{5}{8}\right)$

3. $\dfrac{\frac{1}{14}}{\frac{1}{6} - \frac{1}{7}}$