CS430 Computer Architecture

Spring 2015

Chapter 10
Computer Arithmetic

- Read Chapter 9, much of which is review, and also read Chapter 10.
- We now start to dig into the Central Processing Unit
- We will assume that the memory word size is 8-bits for the purposes of discussion. You should be able to apply the concepts to memory cell sizes of n-bits.

Unsigned Numbers (Modulo 2^n)

1. How many different unsigned numbers can be represented in 8-bits?

2. Write out the bit pattern for the smallest and largest unsigned numbers

Intel Processors

- Use modulo 2^n to represent unsigned integers
- In C++ when an integer is declared as unsigned int x; x is modulo 2^n

1. How many bytes are allocated to an unsigned int?

Signed Numbers

- Signed Magnitude
- Signed magnitude numbers provide for both positive and negative integers. The leftmost bit (MSb - most significant bit) is the sign bit.
- A zero signifies a positive integer and a one signifies a negative integer.
- For example, 00001111 represents a +15 and 10001111 represents a -15.

Signed Magnitude

1. What is the range of integers that can be represented in 8-bits?
2. What is the general formula for this range using N-bits?
3. How many representations of zero exist?
4. What is the representation of -127.
5. If we add one to the value in Q#4, what do we end up with?

Signed Magnitude

- Show the 8-bit pattern for the smallest number, the number on either side of zero and the largest number.

1’s Complement

- One's complement numbers provide for both positive and negative integers.
- The MSb is the sign bit. A zero signifies a positive integer and a one signifies a negative integer.
- For example, 00001111 represents a +15 and but 10001111 does not represent a -15.
- How do we find out what 10001111 represents? The answer is that we invert the bits, add up the positional values of each bit, and put a negative sign in front of the number.

1’s Complement

1. What is the range of integers that can be represented in 8-bits?
2. What is the general formula for this range using N-bits?
3. How many representations of zero exist?
4. What is the representation of -127.
5. If we add one to the value in Q#4, what do we end up with?

1’s Complement

- Show the 8-bit pattern for the smallest number, the number on either side of zero and the largest number.

2’s Complement

- Two's complement numbers provide for both positive and negative integers.
- The MSb is the sign bit. A zero signifies a positive integer and a one signifies a negative integer.
- For example, 00001111 represents a +15 and but 10001111 does not represent a -15.
- In order to find out what this bit pattern represents, take the one's complement and add 1. Sum up the bits and convert to decimal. Finally, place a negative sign in front of the number.

2's Complement

1. What is the range of integers that can be represented in 8-bits?
2. What is the general formula for this range using N-bits?
3. How many representations of zero exist?
4. What is the representation of -127.
5. If we add one to the value in Q#4, what do we end up with?
6. Show the 8-bit pattern for the smallest number, the number on either side of zero and the largest number.

General Problems

1. What is the smallest integer that can be represented in a 12-bit two's complement format?
2. Consider the number $(9A)_{16}$. Is it positive or negative, if it represents an 8-bit two’s complement integer? What is the value represented in decimal?
3. What is the 8-bit two's complement format for $(-27)_{10}$
4. If 01101010 is an 8-bit two's complement integer, what is its decimal equivalent?

General Problems

5. What base 10 value does 1011 represent? Give the answer for the following representation systems. Assume each system uses 4 bits.
   a. UB (unsigned binary)
   b. 1C (one's complement)
   c. 2C (two's complement)
   d. SM (signed magnitude)

General Problems

6. Translate -4_{10} to the following representations. Assume 4 bits. Answers using fewer or more than 4 bits will not be considered correct. If it's not possible to represent -4 in a given representation system, write NR (not representable).
   a) UB (unsigned binary)
   b) 1C (one's complement)
   c) 2C (two's complement)
   d) SM (signed magnitude)

General Problems

7. Using 6-bit binary numbers, represent the decimal number -18 in :
   a. two's complement representation
   b. signed magnitude representation
8. What is the value in decimal of the most negative 6-bit 2’s complement integer?
9. What is the value in decimal of the most positive 6-bit unsigned integer?