## Hash Tables

http://fscked.org/writings/225notes/week13/week13.html
http://en.wikipedia.org/wiki/Hash_table

## Hash Table

- A hash table (or hash map) is a data structure that maps keys (identifiers) into a certain location (bucket)
- A hash function changes the key into an index value (or hash value)

The Hash Table has a fixed length. We'll see how to add space dynamically later.


## Collisions

- Perfect Hash - each key maps to an empty bucket
. Rare!
- Collisions occur where two different keys map to the same bucket
hash(Ryan, D) = 01
hash(Knuth, D) = 01
. Solution?


## Hash Function

- Hash function - compute the key's bucket address from the key
- some function $h(K)$ maps the domain of keys $K$ into a range of addresses $0,1,2, \ldots \mathrm{M}-1$

The Problem

- Finding a suitable function $h$
- Determining a suitable M
- Handling collisions


## Hash Function

- Mid Square
- (turn the key into an integer)
- square the key
- take some number of bits from the center to form the bucket address


## Example

- Problem: Let's assume that the key value is simply the sum of the ASCII values squared. If the key value is 16 -bits and we take the middle 8-bits:
a) How big is the hash table?
b) What is the range of bucket addresses?
c) Where does the key $A B$ map to in the hash table?


## Implementation

. How do we access the middle 8 in an integer?

- // assume 4 byte integers

One Hex-digit is 4 bits unsigned int key $=0 \times 1231 a 456$; unsigned int middle;
middle $=($ key \& 0x000ff000) >> 12; printf("\%08x \%08x\n", key, middle);
pad with zero
8 wide hexadecimal output

## Hash Function

- Division Hashing
- bucket = key \% N
- $N$ is the length of the hash table AND a prime number
a) How big is the hash table?
b) What is the range of bucket addresses?
c) Where does the key $A B$ map to in the hash table?


## Collision Handling

- Open Addressing
- If both K and C map to the same bucket we have a collision
- K and C are distinct
- $K$ is inserted first
- To resolve using OA, find another unoccupied space for C BUT: We must do this systematically so we can find C again easily!
- Analysis: (summation of the \# of probes to locate each key in the table) / \# of keys in the table


## Open Addressing

- Find another open bucket
- bucket $=(\mathrm{h}(\mathrm{K})+\mathrm{f}(\mathrm{i})) \% \mathrm{~N}$
. $N$ is the length of the table
- $\mathrm{h}(\mathrm{K})$ : original hash of key K
- $f(i)$ : $i$ is the number of times you have hashed and failed to find an empty slot
- First hash is:
- bucket $=(h(K)+f(0)) \% N$
$-f(0)=0$


## Linear Probing

- $\mathrm{f}(\mathrm{i})=\mathrm{i}$
- Example:
$h(K n)=n \% 11$
Insert
M13
G7
Q17
Y25
R18
Z26

| Bucket | Data |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 | 10 |

F6
Primary Clustering!

## Primary Clustering

- Primary Clustering - this implies that all keys that collide at address $b$ will extend the cluster that contains b


## Quadratic Probing

- $\mathrm{f}(\mathrm{i})=\mathrm{i} \mathrm{i}^{\wedge} 2$
- Example: $h(K n)=n \% 11$
Insert
M13
G7
Q17
Y25
R18
Z26

| Bucket | Data |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

Secondary Clustering!

## Secondary Clustering

- Secondary Clustering - is when adjacent clusters join to form a composite cluster


## Double Hash

- $f(i)=h 2(k)$ * $i$
- h2(k) is some second hash function
- unique probe sequence for every key
- bucket $=(\mathrm{h}(\mathrm{K})+\mathrm{h} 2(\mathrm{~K}) * \mathrm{i}) \% \mathrm{~N}$
- $\mathrm{h} 2(\mathrm{k})$ should be relatively prime to N for all k
- don't produce zero
- Example
$-h(k)=k \% N$
$h 2(k)=1+(k \%(N-1))$


## Rehash

- Reallocate the table larger and reinsert every element


## Chaining (Open Hashing)

- Each bucket is the head of a linked list
- if you hash a key to a bucket, insert the data into the list
- insert at front, back, or in sorted order.
- why would this decision matter?


## Problem

. Hash the keys M13, G7, Q17, Y25, R18, Z26, and F6 using the hash formula $h(K n)=n \bmod 9$ with the following collision handling technique: (a) linear probing, (b) chaining

- Compute the average number of probes to find an arbitrary key K for both methods.
- $\operatorname{avg}=$ (summation of the \# of probes to locate each key in the table) / \# of keys in the table

