More BSTs & AVL Trees

bstDelete

```
if (key not found)
  return
else if (either subtree is empty)
{
  delete the node replacing the parents link with the
  ptr to the nonempty subtree or NULL if both
  subtrees are empty
}
else
{
  Traverse the left subtree of the node to be deleted
    such that you find the rightmost node (Rnode) in the left
    subtree

  Move the contents of Rnode to the node to be deleted

  Set Rnode's parent pointer to point to the left subtree
    of Rnode

  Free the unused node
}
```

bstDelete

- Create a BST from the following keys: 10, 5, 15, 2, 8, 12, 7, 16, 14

- Assume that you always start with the above tree, how would each of the following keys be deleted?
  - 10
  - 15
  - 5

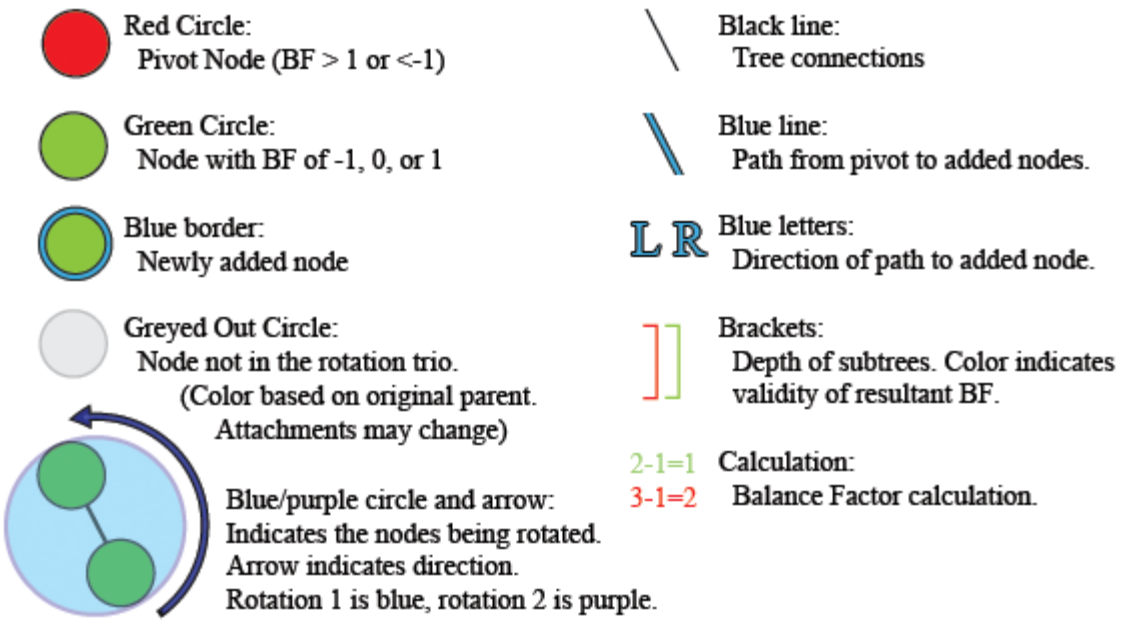
AVL Trees
Adelson-Velskii & Landis

- **Defn**: A binary tree is a height-balanced p-tree if for each node in the binary tree, the difference in the height of the left and right subtrees is at most p.
- **Defn**: An AVL (Adelson-Velskii, Landis) tree is a binary search height-balanced 1- tree.
- **Defn**: The balance factor of a node, BF(node), in a binary tree is the difference of the left and right subtrees, hL - hR.
- For any node in an AVL tree, the balance factor is either -1, 0, or 1.

AVL Trees

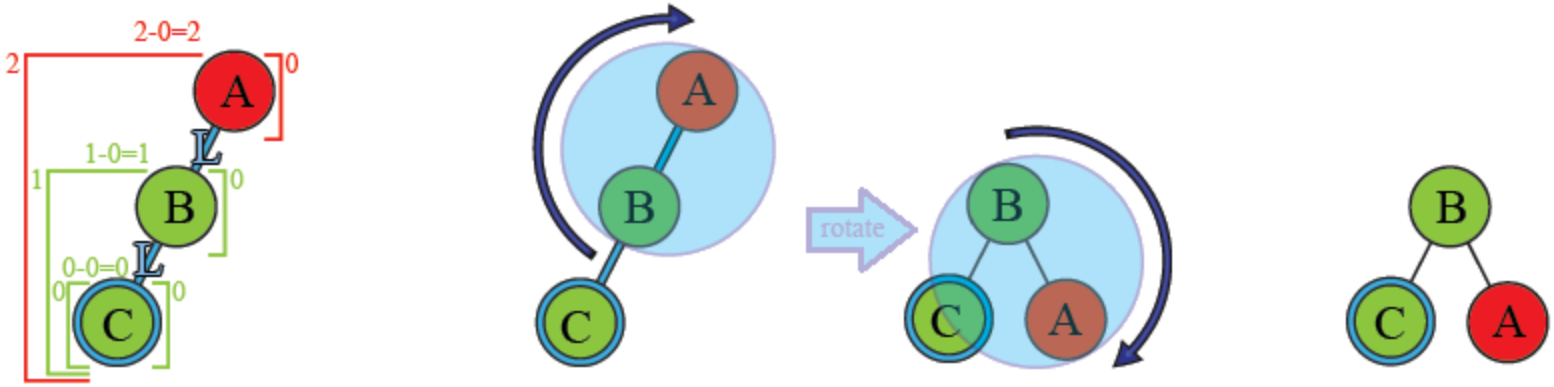
- After inserting a new value into an AVL tree, if any node has a BF other than -1, 0, or 1, the AVL tree must be rebalanced.
- The AVL tree is rebalanced at the closest ancestor, of the inserted node, that has a BF of -2 or +2. We will call the closest ancestor with a BF of +2 or -2 of the inserted node the pivot node, P.
- Four basic rotations are possible where two are single rotations and two are double rotations.

AVL Trees

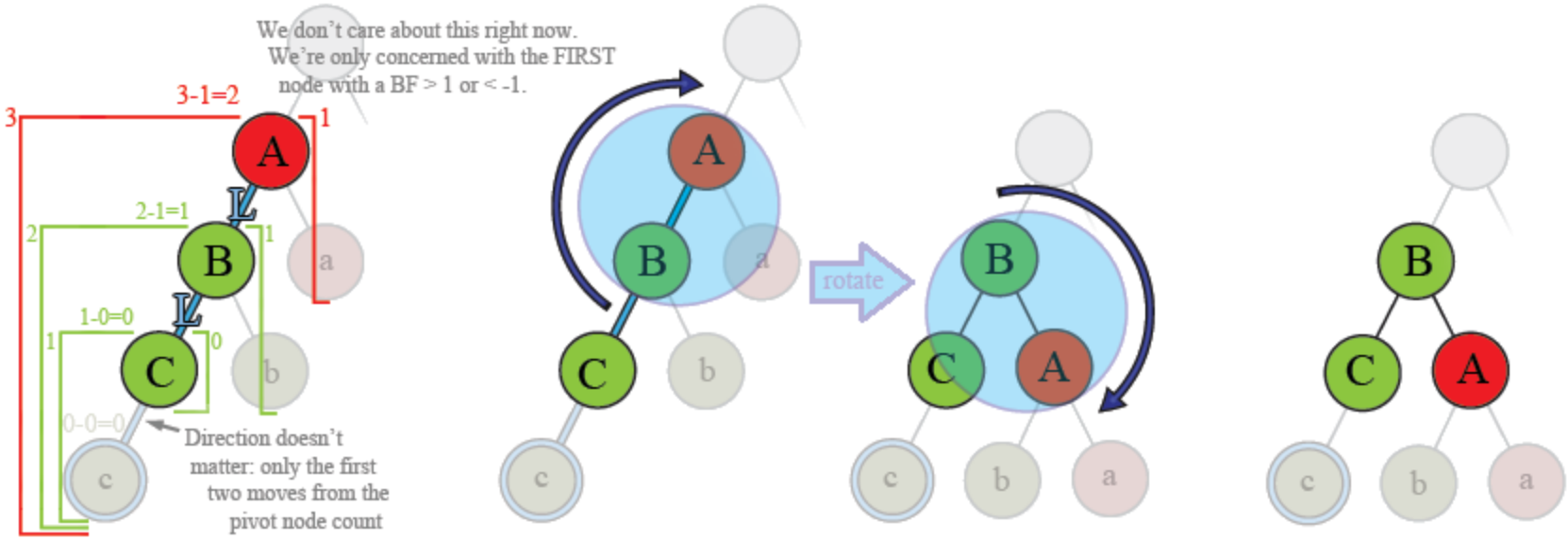


The following diagrams were made by Alex Shinsel

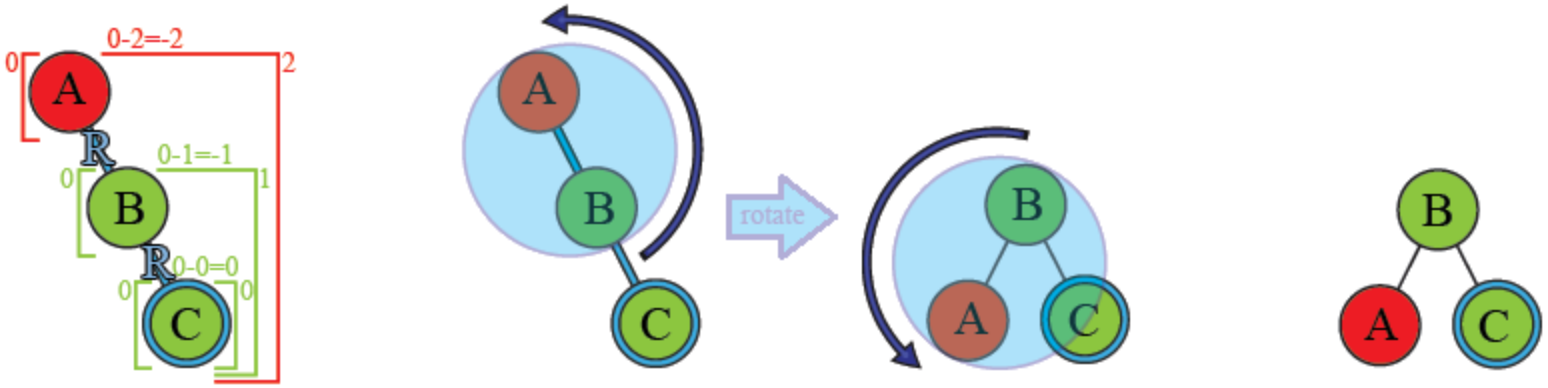
AVL Trees
LL Rotation



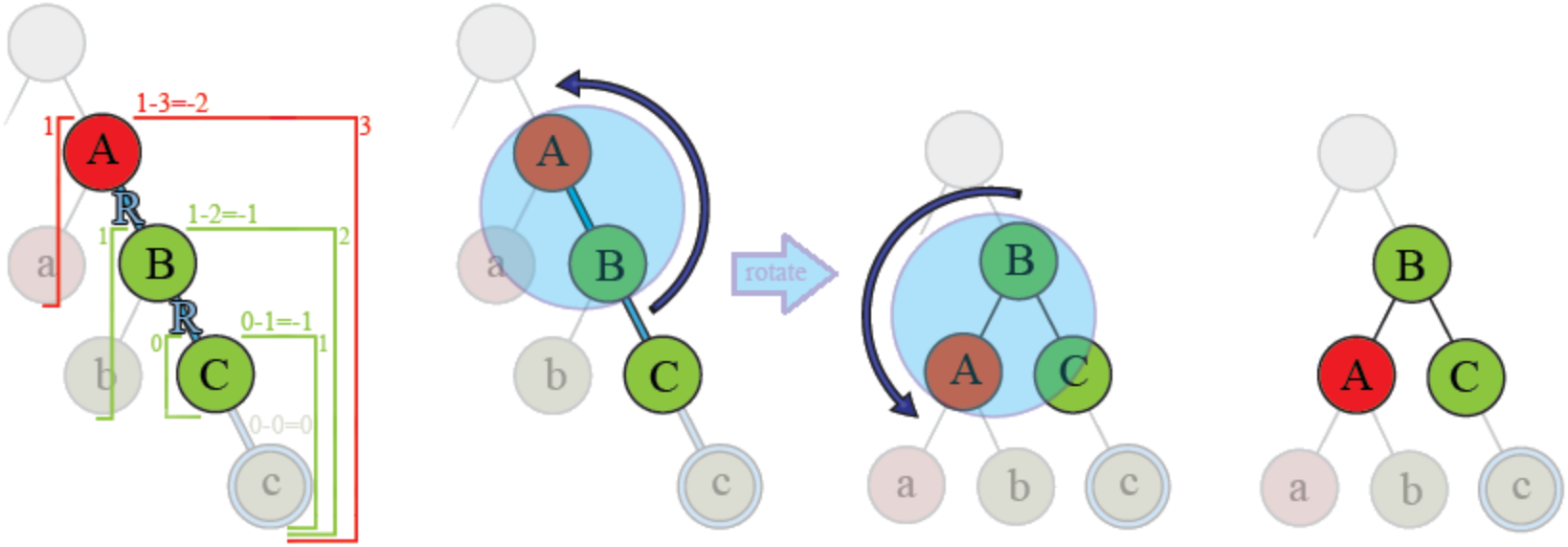
AVL Trees
LL Rotation



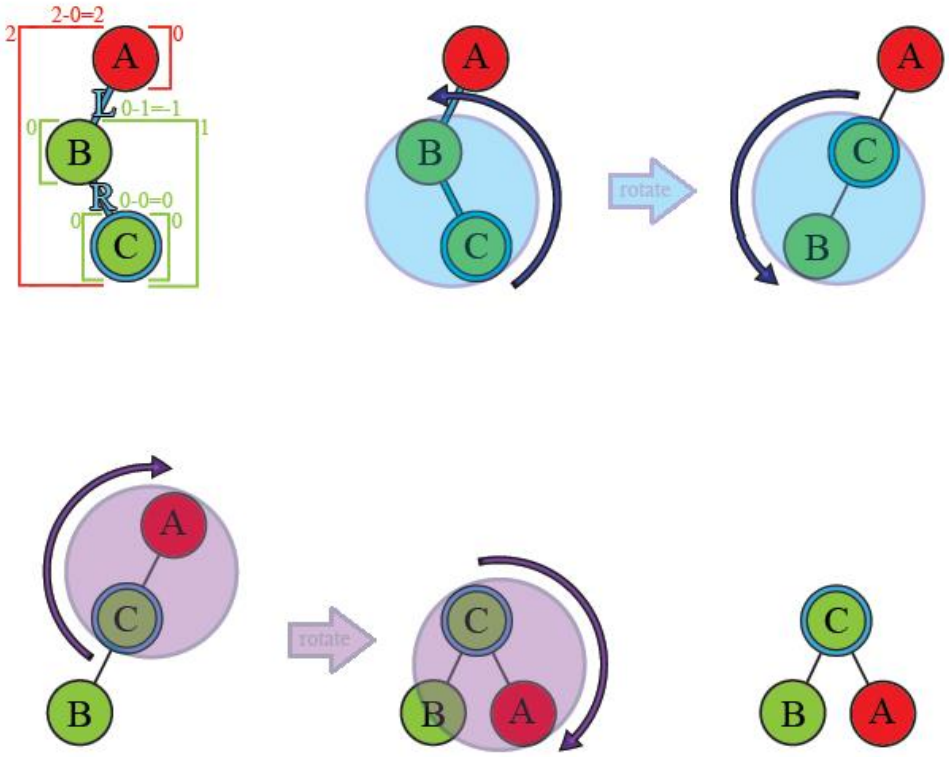
AVL Trees
RR Rotation



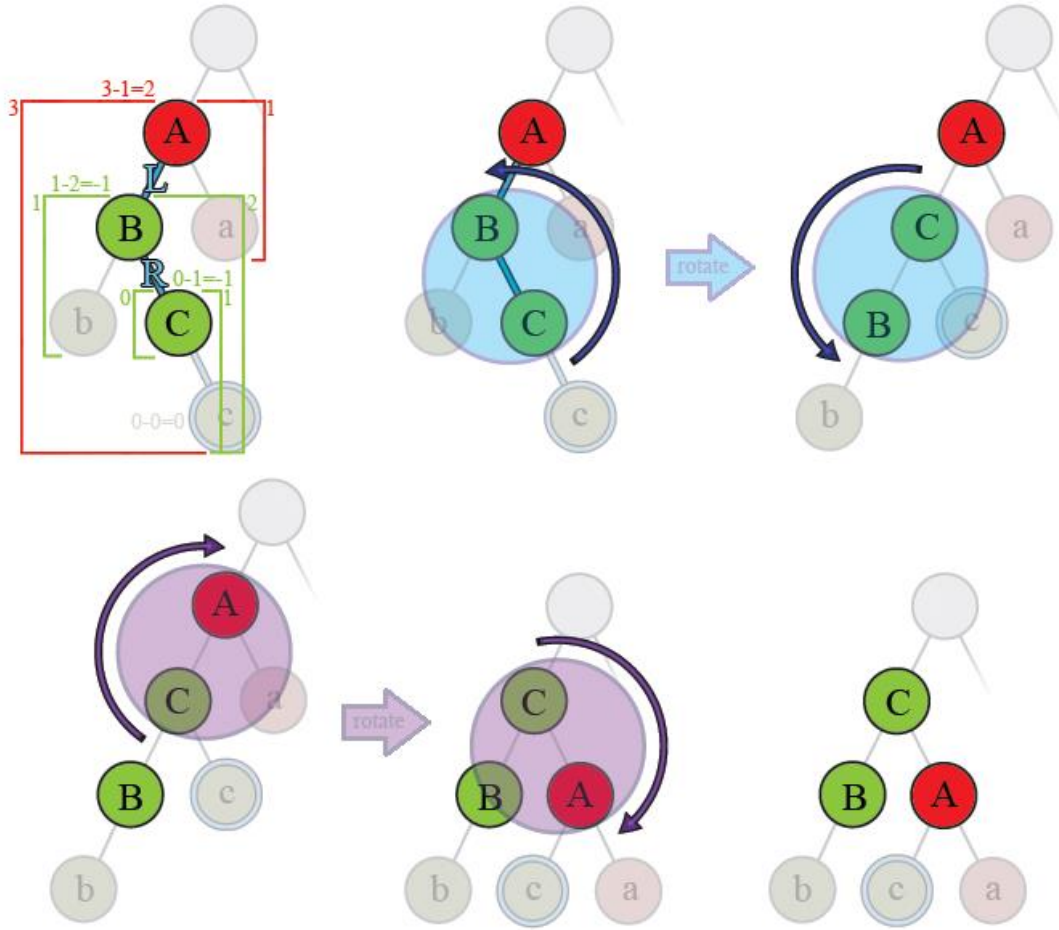
AVL Trees
RR Rotation



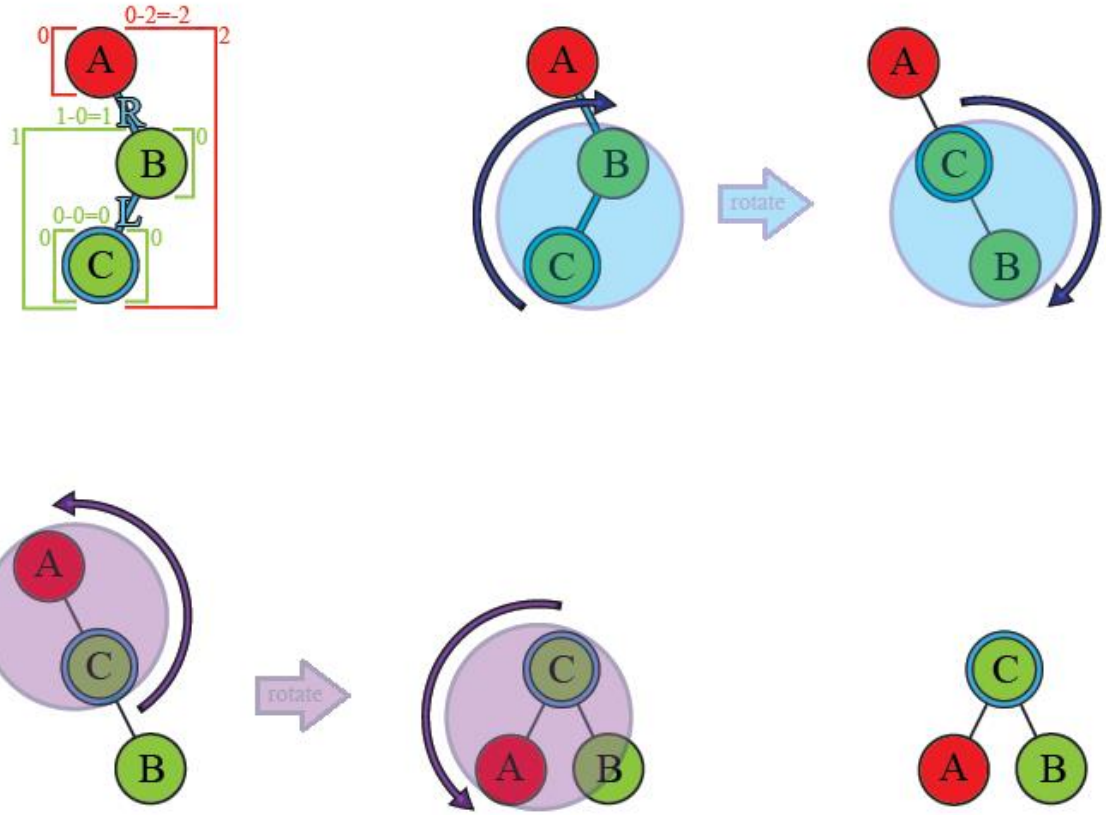
AVL Trees
LR Rotation



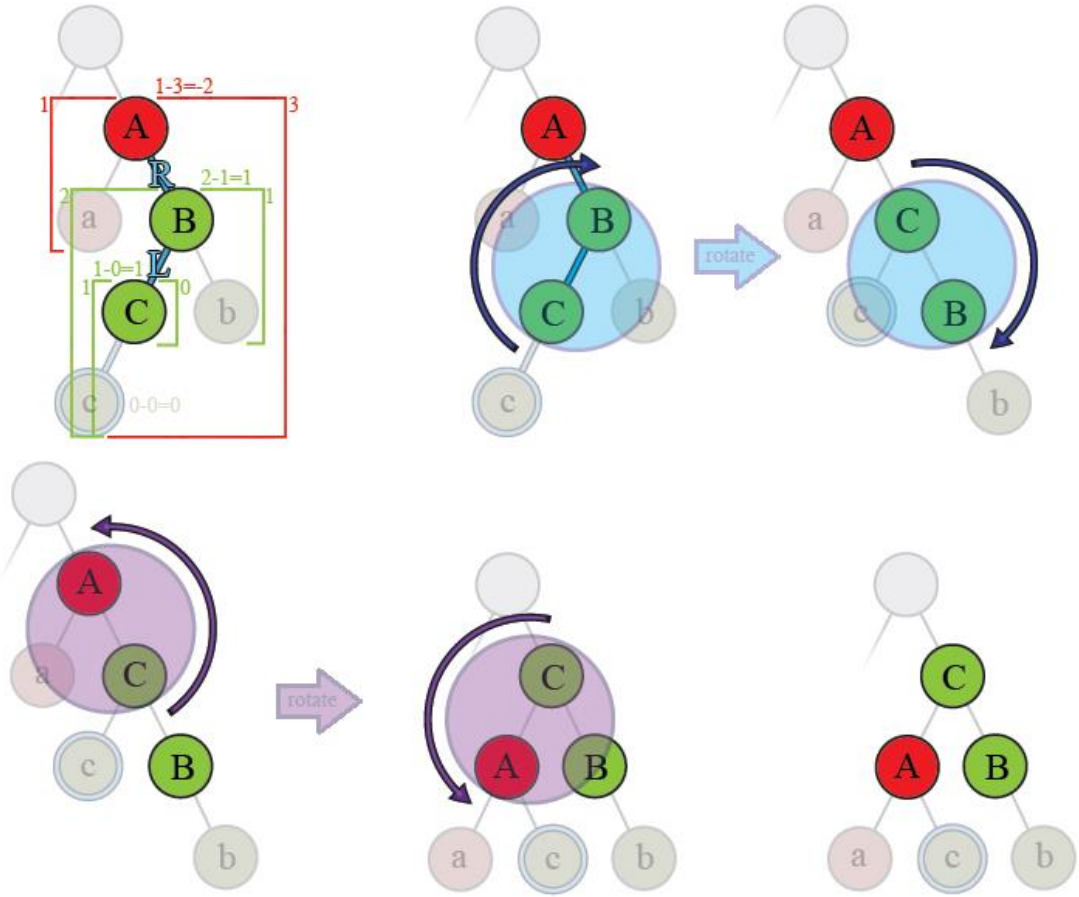
AVL Trees
LR Rotation



AVL Trees
RL Rotation

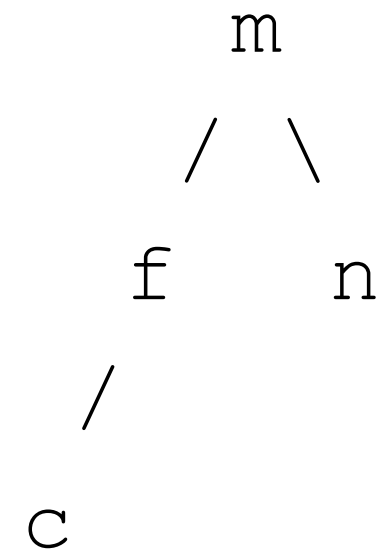


AVL Trees
RL Rotation



Problems

- Consider



Q1: Is the tree an AVL tree? Why or why not?

Q2: Does the tree need any kind of rebalancing? If so, rebalance the tree.

P1: Insert **z** into the tree.

Q3: Does the resulting tree need rebalancing? Why or why not? If so, rebalance the tree.

P2: Insert **a** into the tree.

Q4: Does the resulting tree need rebalancing? Why or why not? If so, rebalance the tree.

P3: Starting over, insert **j and g** into the tree. Rebalance when necessary.

P4: Starting over, insert **j and a** into the tree. Rebalance when necessary.

Problems

- Insert the following months into an AVL tree lexicographically: Mar, May, Nov, Aug, Apr, Jan, Dec, Jul, Feb, Jun, Oct, Sep

- If a rebalance is needed, show the proper rebalance notation for the type of rebalance applied to the AVL tree