Finite Automata
Sections: 1.1, 1.2
September 9, 2016
Lecture 5

Quick Review

• Deterministic Finite Automata:
  5-tuple (Q, ∑, δ, q₀, F)
  Q: finite set of states
  ∑: alphabet (finite set)
  δ: transition function (δ: Q × ∑ → Q)
  q₀: start state
  F: accepting states (subset of Q)

• Language A is regular if there exists a Finite Automata that recognizes A.
Regular Language

- Determinism?

- Regular language
  - Example?

  - Example of non-regular language?

Regular Operations on Languages

- Given two languages, A,B, we can create new languages in a variety of ways:
  - What operations have we seen?
$\Sigma = \{0,1\} \quad A = \{w \mid w \text{ ends in } 1\} \quad \text{Examples}$

$B = \{w \mid w \text{ begins with } 00\}$

$A \cup B = \quad AB = \quad A^* = \quad A \cap B = \quad \overline{A} =$

**Closure of Regular Languages**

• A set is **closed** under some operation, Examples?

• **Regular operations**
Proof

• Theorem 1.25: The class of regular languages is closed under the union operation.

• If A and B are regular languages, so is $A \cup B$

  What do we need to prove?
  What does regular mean?
  What does it mean for $A \cup B$ to be regular?
\[ \Sigma = \{0, 1\} \]
\[ A = \{ w | \text{w contains a 1 in the penultimate position} \} \]

A = \{
\}

Nondeterminism

Nondeterministic Finite Automata:
- Can have none, one, or many exiting arrows for each state and each letter of alphabet
- Often running several processes at once!
NFA

E-transitions
• performed automatically without any input!
• when entering state with ε-transition, simultaneously begin running two processes:
  • one stays in current state
  • second follows ε-transition

Why would we ever use this?

Example

• Does this NFA accept 010110?
• What sequence of states does it go through?
\[\Sigma = \{0, 1\}\]

**Build the machine**

\[A = \{w \mid \text{w contains a 1 in the penultimate position}\}\]

**Proof**

- Theorem 1.26: The class of regular languages is closed under the concatenation operation.

- If A and B are regular languages, so is AB.

What do we need to prove?
What does regular mean?
What does it mean for AB to be regular?
Problems?
### Examples

A = \{\text{north, northern}\} \quad B = \{\text{east, west}\}

\(s = \text{northerneast}\) is in AB

Are A and B regular languages?

A = \{w \mid w \text{ begins with 1 ends with 0}\}
B = \{w \mid w \text{ begins with 0 ends with 1}\}

\(s = 1000011\)