Reduction

• Convert one problem (A) into a second problem (B)
  – solution to B can be used to solve A
  – If B is decidable, so is A
  – If A is undecidable, so is B

– Is Z undecidable? Prove it is reducible to Y, which has previously been shown to be undecidable
Halting Problem

$\text{HALT}_{TM} = \{ <M,w> | M \text{ is a TM and } M \text{ halts on input } w \}$

undecidable?

• Proof: Assume $\text{HALT}_{TM}$ is decidable, show that if true, $A_{TM}$ is decidable.
  – Contradiction!

• $A_{TM}$ is reducible to $\text{HALT}_{TM}$

Proof

• Assume TM R decides $\text{HALT}_{TM}$
  • Use R to build TM S that decides $A_{TM}$

  • S: Run TM R on $<M, w>$
    – If R rejects, reject
    – If R accepts, run M on w until M halts
  • If M accepts, accept, if M rejects, reject
  • If R decides $\text{HALT}_{TM}$ then $A_{TM}$ is decidable
  • $A_{TM}$ is reducible to $\text{HALT}_{TM}$
**TM Equality**

• \( \text{EQ}_{\text{TM}} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2) \} \)

• \( \text{E}_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \} \)
  – undecidable (see TH 5.2 p 189)

• Show that if \( \text{EQ}_{\text{TM}} \) were decidable, so would be \( \text{E}_{\text{TM}} \)
• Reduction from \( \text{E}_{\text{TM}} \) to \( \text{EQ}_{\text{TM}} \)
  – \( \text{E}_{\text{TM}} \) is a special case of \( \text{EQ}_{\text{TM}} \) where \( L(M_i) = \emptyset \)

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**Computation Histories**

• List of configurations a TM goes through

• Configuration
  – Current State
  – Current Tape State
  – Read/Write Head location

• Finite sequence that ends in accept or reject
Linear Bounded Automaton

- Cannot move read/write head off portion of tape with original input

- May have larger tape alphabet than input alphabet
  - Allows for larger memory than just number of tape positions
  - Increase by constant factor

Proof

- $A_{LBA} = \{<M,w> | M is an LBA that accepts string w\}$

- Decidable
  - Proof using computation histories
  - LBA with $q$ states, $g$ symbols in tape alphabet, input tape of length $n$
  - How many possible configurations are there?
  - ???