CS310

Variants of Turing Machines

Section 3.2

November 4, 2016

Review: Formal Definition

• 7 Tuple:
Multiple Tape Turing Machine

• For k tapes
  – input string is on tape 1
  – other tapes start out blank

• New transition function (S = STAY)
  \[ \delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k \]

• Example (for 2-tape)
  \[ \delta (q_1,0,1) \rightarrow (q_2,1,0,L,R) \]

Example

• Construct a two-tape Turing Machine to accept \( L = \{a^n b^n \mid n \geq 1\} \)

• Conceptually what do we want to do?

• Recognizer or Decider?
**Theorem**

- Every multi-tape Turing Machine has an equivalent single tape machine
  – *adding extra tapes does not add power to the Turing Machine*
- Proof Idea: Simulate multi-tape TM as single tape TM

**Nondeterministic TM**

- Often easier to design/understand
- Transition function becomes
  \[ \delta : Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\}) \]
- We can think of this computation as a tree
  – each branch from a node (state) represents one nondeterministic decision (for a single input character).
Nondeterministic TM

• Design a TM to accept strings containing a $c$ that is either preceded or followed by $ab$

• Idea: Stay in initial state $q_0$ until a $c$ is encountered. When occurs, stay in state $q_0$ AND enter state $q_1$ to determine if $c$ is followed by $ab$, or enter state $q_3$ to determine if $c$ is preceded by $ab$

• NOTE: JFLAP doesn’t support N-TM’s.
Theorem

• Every nondeterministic TM, N, has an equivalent deterministic TM, D

• Proof Idea:
  – use a 3 tape TM (we can convert this to a one tape TM later)
  – tape 1: input tape (read-only)
  – tape 2: simulation input/output tape of the current branch of the n-d TM
  – tape 3: address tape (based on the tree) to keep track of where we are in the computation

Practice

{ a^i b^j c^k | i > j > 0; k = 2i }
{ ww^R | | ww^R | is odd , w ∈ {0,1}^* }  
{ ww | w ∈ {0,1}^* }  
the complement of {ww^R | w ∈ {0,1}^* }

multiplication of two numbers in base 1:
11111 * 11 produces 1111111111