Pumping Lemma
Section: 1.4
September 26, 2016
Lecture 10

String Review

• w is a string
• |w| is

• ww means
• w^n means

• w = xyz, x is a ____________ of w
  
z is a ____________ of w
Non-Regular Languages

• Languages that cannot be represented by a finite automaton
  – Such as?

• How do we prove a language is not regular?
  – What characteristics must a language have to be regular?

  \[ C = \{ w \mid w \text{ has an equal number of 0s and 1s} \} \]
  \[ D = \{ w \mid w \text{ has an equal number of occurrences of 01 and 10 as substrings} \} \]

Pumping Lemma (Informal)

Pumping: The length of the string could be ‘pumped’ up by repeating a cycle in the FA, and the string would still be accepted.

• All regular languages have a property
  – the pumping length, \( p \)

• \( |w| = n \), how many states do we go through?
Pumping Lemma (Formally)

- DFA: \( M = (Q, \Sigma, \delta, q_0, F) \)
- If \(|Q| = p\) and \(s \in L(M)\) and \(|s| \geq p\) then there exists at least one state that was visited twice within the first \(p\) input symbols

\[ s = xyz \]

\( p \) – pumping length

- For every regular language, some integer \(p\) exists.
- We do not care what the actual integer value of \(p\) is
- We will always refer just to \(p\)
Pumping Lemma (Formally)

• If A is a regular language, then:

\[ s = xyz \]

\[ i \geq 0, \ xy^iz \in L(M) \]

\[ |y| > 0 \quad (x, z \text{ may be } \epsilon) \]

\[ |xy| \leq p \]

Pumping Lemma In Action

• Find a string, \( s \in L, |s| \geq p \), that cannot be pumped to show language L is not regular.

– Find a string that exhibits the “essence” of nonregularity

– **Hint:** choose a string that explicitly references the value p!

– Proof method?

• \( L = \{ w \mid w \text{ contains equal number of 0s and 1s} \} \)
Practice

•\( L = \{ \text{ww} \mid w \in \{0, 1\}^* \} \)

What string should we chose?

what does \text{ww} mean?

Can that be pumped?

Regular vs Non-Regular

\( L = \{ 1^n \} \quad \Sigma = \{0, 1\} \)

\( L = \{ 1^n0^n \} \)

\( L = \{ 1^n \mid n \geq 0 \} \)

\( L = \{ 0^n1^n \mid n \geq 0 \} \)
Examples Galore!

- $L = \{ a^n b^m \mid m > n \}$
- $L = \{ a^n b^m \mid m \text{ is odd}, n \text{ is even}, m>0, n>0 \}$
- $L = \{ w1w^R \mid w \in \{0,1\}^* \}$
- $L = \{ a^n b^m \mid m \neq n \}$
- $L = \{ a^{2n} \mid n > 0 \}$
- $L = \{ a^n \mid n \text{ is prime} \}$
- $L = \{ a^n b^m c^{n+m} \mid n, m > 0 \}$
- $L = \{ w^R \mid w \in \{0,1\}^*, w \text{ is a perfect square in binary} \}$
- $L = \{ wbbw \mid w \in \{a, b\}^* \}$
- $L = \{ (ac)^n b^m \mid n > m >= 0 \}$
- $L = \{ a^n b^m \mid m > 2, n > 2 \}$

Show for each language:
- Are any of these languages regular?
- Can we write any of them as a regular expression?

Practice

- $L = \{ w \mid 1^n0^m1^n, n > 0, m >=0 \}$ Is $L$ regular?
- Which of the following strings are in $L$ and do not violate the pumping lemma?

- $s=10^p1$  \hspace{2cm} $x =$
- $s=1^2p$  \hspace{2cm} $y =$
- $s=1^p0^p1^p$  \hspace{2cm} $z =$
- $s=0$  \hspace{2cm} $xy^iz$
- $s=1^p0^p1^p$  \hspace{2cm} $i >=0$
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<th>More Practice</th>
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<td>( L = \left{ w \mid 1^n0^m1^n, 0 &lt; n &lt; 4, m \geq 0 \right} ) Is ( L ) regular?</td>
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