CS310

NP-Completeness

Section 7.4

November 30, 2014

NP-Complete

- NP-Completeness
  - set of problems in NP whose complexity is related to all problems in NP
  - if an NP-Complete problem can be shown to be in P, then P=NP
  - boolean satisfiability, for example
  - vertex-cover
  - clique
  - Hamilton Path

Boolean Satisfiability

And
Or
Not
Like a circuit

- Is a boolean formula satisfiable?
  - Does some set of values produce true?

$$\phi = (\bar{x} \vee y \vee z) \wedge (x \vee \bar{z} \vee y) \quad \bar{z} \text{ means } \neg z$$

SAT = { <ϕ> | ϕ is a satisfiable Boolean formula}

  - Clause
  - Conjunctive normal form (cnf)
  - 3cnf

3SAT = { <ϕ> | ϕ is a satisfiable 3cnf Boolean formula}

  - Cook-Levin Theorem: SAT $\in$ P iff P = NP

Reducibility

- If problem A is *efficiently* reducible to problem B, an efficient solution to B can be used to solve A *efficiently*

  A function *f* is a *polynomial time computable function* if some polynomial time TM exists that halts with just f(w) on the tape when run on input w.

Cont.

- Language A is polynomial time reducible to language B, $A \leq_p B$, if a polynomial time computable function f exists where for every w:

  $w \in A \Leftrightarrow f(w) \in B$

3SAT reduces to CLIQUE

- Polynomial time reduction
- If CLIQUE is in P, so is 3SAT

- Turn 3SAT into a graph
  – Identify a CLIQUE to find a solution to 3SAT

NP-Complete

- B is NP-Complete if:
  - B is in NP
  - Every A in NP is polynomial time reducible to B
- If B is NP-Complete, and $B \in P$, then P=NP

SAT is NP-Complete

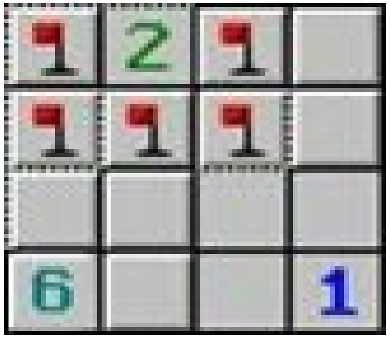
- SAT is in NP
- Show that every language in NP can be

  polynomial time reduced to SAT

Minesweeper is NP-Complete

- Given a partial board, is it a valid Minesweeper board?

  Can convert SAT problem into a Minesweeper board.

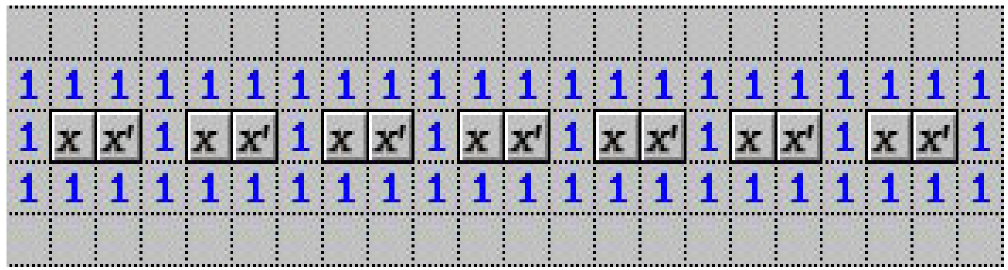


Invalid Board

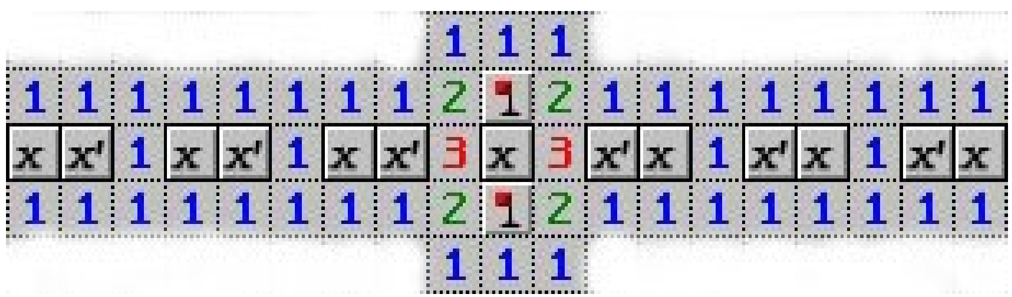
http://www.claymath.org/Popular_Lectures/Minesweeper/

Build the board from SAT

- Cell with mine is True



Wire to propagate a value

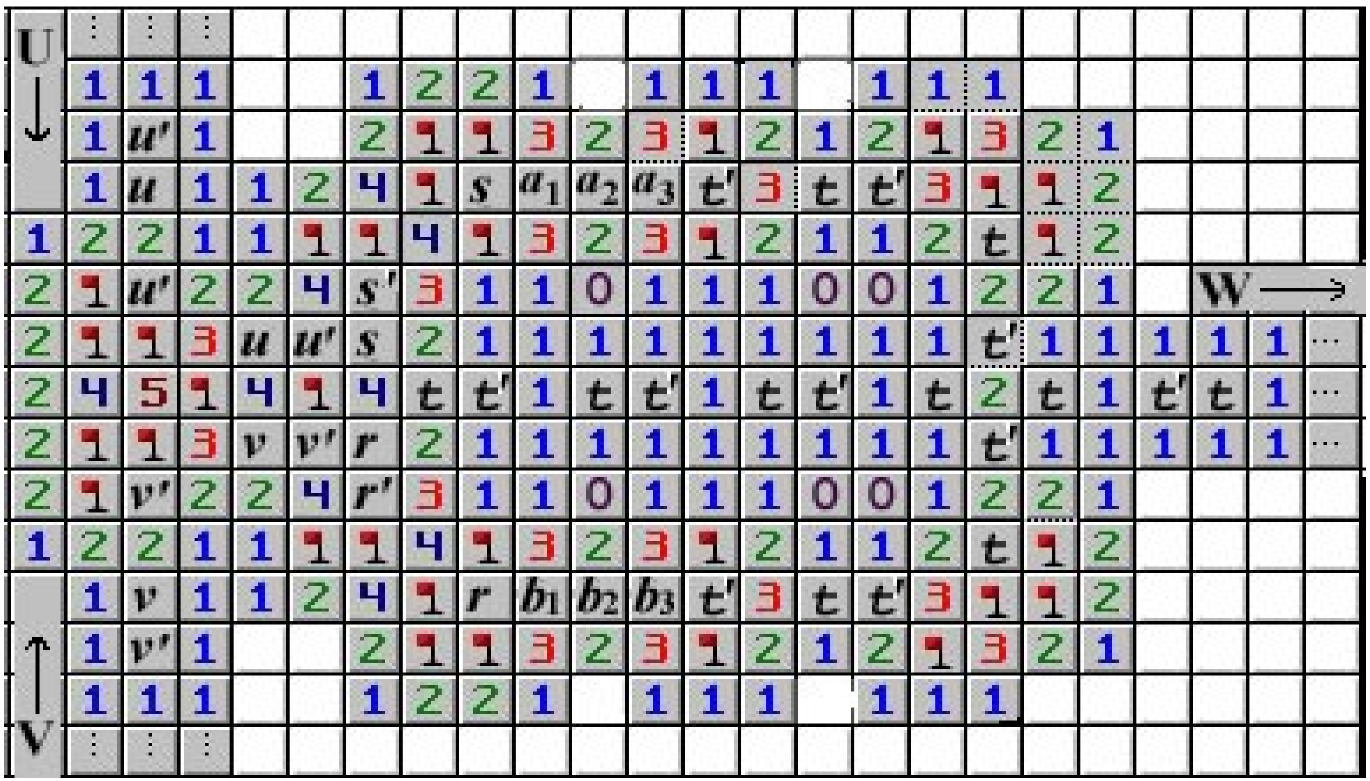


Not Gate

http://www.claymath.org/Popular_Lectures/Minesweeper/

Cont.

- And Gate
  U & V are input wires, W is output Wire



What about OR?

http://www.claymath.org/Popular_Lectures/Minesweeper/