CS310

Reducibility

Chapter 5.1

November 26, 2012

Reduction

- Convert one problem (A) into a second problem (B)
  - solution to B can be used to solve A
  - If B is decidable, so is A
  - If A is undecidable, so is B

  - Is Z undecidable? Prove it is reducible to Y, which has previously been shown to be undecidable

Halting Problem

$HALT_{TM} = \{<M,w>|M \text{ is a TM and } M \text{ halts on input } w\}$

undecidable?

- Proof: Assume $HALT_{TM}$ is decidable, show that if true, A_{TM} is decidable.
  - Contradiction!
- A_{TM} is reducible to $HALT_{TM}$

Proof

- Assume TM R decides $HALT_{TM}$
- Use R to build TM S that decides A_{TM}

- S: Run TM R on <M, w>
  - If R rejects, reject
  - If R accepts, run M on w until M halts
    - If M accepts, accept, if M rejects, reject
- If R decides $HALT_{TM}$ then A_{TM} is decidable
- A_{TM} is reducible to $HALT_{TM}$

TM Equality

- $EQ_{TM} = \{<M_1, M_2> \mid M_1$ and M_2 are TMs and $L(M_1) = L(M_2)\}$
- $E_{TM} = \{<M> \mid M$ is a TM and $L(M) = \emptyset\}$
  - undecidable (see TH 5.2 p 189)
- Show that if EQ_{TM} were decidable, so would be E_{TM}
- Reduction from E_{TM} to EQ_{TM}
  - E_{TM} is a special case of EQ_{TM} where $L(M_i) = \emptyset$

Computation Histories

- List of configurations a TM goes through

- Configuration
  - Current State
  - Current Tape State
  - Read/Write Head location

- Finite sequence that ends in accept or reject

Linear Bounded Automaton

- Cannot move read/write head off portion of tape with original input
- May have larger tape alphabet than input alphabet
  - Allows for larger memory than just number of tape positions
  - Increase by constant factor

Proof

- A_{LBA} = {<M,w> | M is an LBA that accepts string w}

- Decidable
- Proof using computation histories
  - LBA with *q* states, *g* symbols in tape alphabet, input tape of length *n*
  - How many possible configurations are there?
  - ???