

CS310

Variants of Turing Machines

Section 3.3

November 12, 2014

Definition of Algorithm

- Algorithm: collection of simple instructions for carrying out some task (Sipser p 154)
- Hilbert: 23 mathematical problems for the new century (1900)
- 10th problem: devise algorithm that tests whether a polynomial has an **integral root**
 - no such algorithm exists

Algorithm

- Finding an algorithm: easy
- Proving no such algorithm exists: difficult
 - especially if you can't reason about/discuss an algorithm
- 1936
 - Alonzo Church: algorithm as λ -calculus
 - Alan Turing: algorithm as machines
 - found to be equivalent
- Church-Turing Thesis
 - used to describe limits of computation
 - used on Hilbert's 10th problem

Hilbert's 10th Problem

- $D = \{ p \mid p \text{ is a polynomial with an integral root} \}$
- Hilbert: Is D decidable?
 - No
- But it is Turing-recognizable
 - how could a TM **recognize** this language?
 $6x^3y^2 - 2x - 3y - 1$

Terminology

- We care about algorithms, not TMs
- Descriptions
 - formal
 - full states and transitions
 - implementation
 - English prose describes how the tape is used
 - high-level *
 - English prose to describe an algorithm
 - no implementation details
- Data : $\langle G \rangle$ an encoded piece of data, G
 - any data can be encoded as a string of 1s and 0s