

CS310

# Variants of Turing Machines

Section 3.2

November 10, 2014

# Formal Definition

- 7 Tuple:

# Multiple Tape Turing Machine

- For  $k$  tapes
  - input string is on tape 1
  - other tapes start out blank
- Change

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

# Example

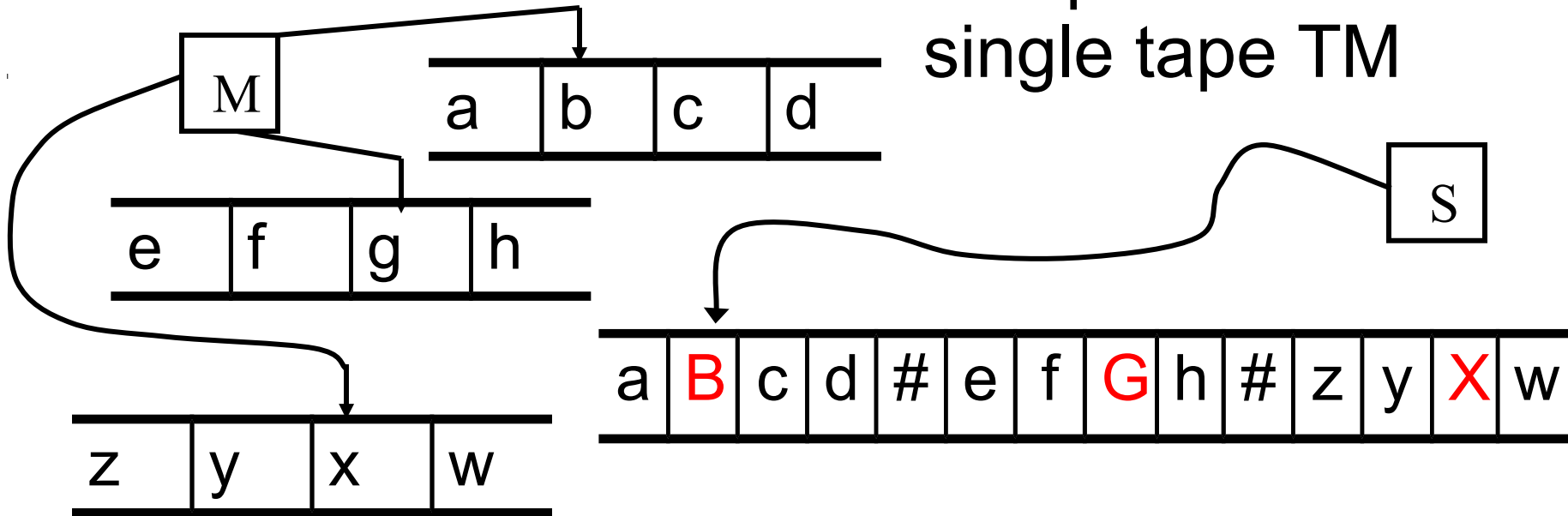
- Construct a two-tape Turing Machine to accept  $L = \{a^n b^n \mid n \geq 1\}$
- Conceptually what do we want to do?

# Theorem

- Every multi-tape Turing Machine has an equivalent single tape machine

– *adding extra tapes does not add power to the Turing Machine*

- Proof Idea: Simulate multi-tape TM as



# Nondeterministic TM

- Often easier to design/understand
- Design a TM to accept strings containing a  $c$  that is either preceded or followed by  $ab$
- We can think of this computation as a tree
  - each branch from a node (state) represents one nondeterministic decision (for a single input character)

# Theorem

- Every nondeterministic TM,  $N$ , has an equivalent deterministic TM,  $D$
- Proof Idea:
  - use a 3 tape TM (we can convert this to a one tape TM later)
  - tape 1: input tape (read-only)
  - tape 2: simulation input/output tape of the current branch of the n-d TM
  - tape 3: address tape (based on the tree) to keep track of where we are in the computation

# Practice

$\{ a^i b^j c^k \mid i > j > 0; k = 2i \}$

$\{ ww^R \mid |ww^R| \text{ is odd}, w \in \{0,1\}^* \}$

$\{ ww \mid w \in \{0,1\}^* \}$

the complement of  $\{ww^R \mid w \in \{0,1\}^* \}$

multiplication of two numbers in base 1:

$11111 * 11$  produces  $1111111111$