CS310

Turing Machines

Section 3.1

November 3, 2014

Alan Turing

- English mathematician
- Helped break German codes during WWII
- Helped formalize the concept of an **algorithm**
  - Represented by the Turing Machine
  - A TM is a precise way to discuss/reason about algorithms
  - Data: any data can be encoded as a string of 0s and 1s

- Similar to Finite Automata

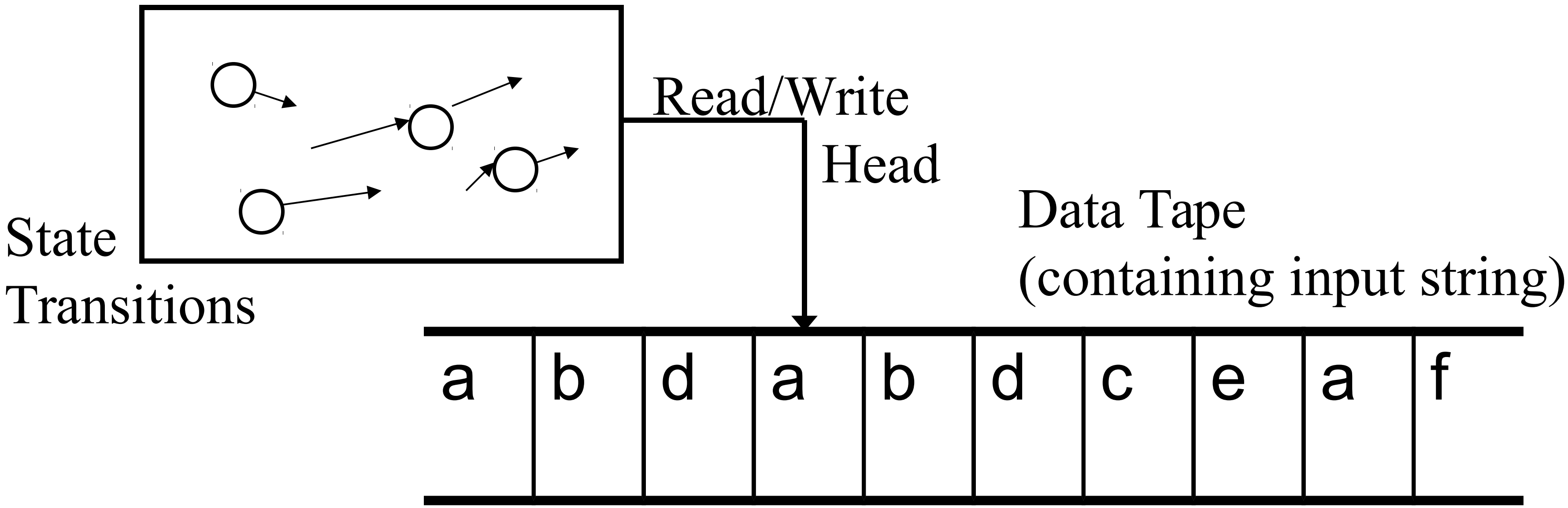
  – Differences
    - unlimited and unrestricted memory

  – more accurate model of modern computer

- Problems that cannot be solved by a Turing Machine cannot be solved by a "real" digital computer
  – theoretical limits of computation

Turing Machine

- State Transitions plus infinite “data tape”
  - read/write tape
  - move around on tape



Notes

- Deterministic

- May make multiple passes over input

- Reject string by entering reject configuration or looping forever
  - hard to tell if a machine will loop forever
  - Halting problem

Differences with FA

- TM can read and write from tape

- Read/Write head can move left or right
  - One step
  - must move

- TM tape is infinite to the right*
- TM accept and reject states take effect *immediately*

Example

- $L = \{ w\#w \mid w \in \{0,1\}^* \}$
- Conceptually, we want to do what?

- input string:

1	1	0	0	#	1	1	0	0	⊔

Is L regular? context free?

Formal Definition (7 Tuple)

- $\{Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}\}$

Q:

Σ:

blank character: ⊔

Γ:

δ:

$q_{accept} \neq q_{reject}$

Operation

- Start configuration of M on input w is:q_0w

- **Yield**: uaq_ibv yields uq_nacv
  if $\delta(q_i, b) \rightarrow (q_n, c, L)$

- Accepting and Rejecting configurations are called *halting* configurations
  - the TM stops operating
  - otherwise, loops forever

Definition of Computing

- A TM, M, accepts a string, w, if there exists a sequence of configurations, $c_0,c_1,\ldots,c_n$, such that:
  - c_0 is the start configuration
  - c_i yields c_{i+1} for all i
  - c_n is an accept configuration
- The set of strings M accepts is L(M)
  - language of M

Definitions

- Turing recognizable
  - a language is Turing Recognizable if some TM recognizes it (accepts all valid strings)
- Turing decidable
  - a language is Turing decidable if some TM decides it
  - halts on all inputs
    - hard to tell if a looping machine is really going to reject the string

Church-Turing Thesis

- Turing Model is and always will be the most powerful model
  - it can simulate other models: D/NFA, PDA
  - variations do not provide more power
    - extra tape
    - nondeterminism
    - extra read/write heads
    - (but may make a TM easier to build)

Build a Machine!

Often, you write an algorithm for the machine rather than a set of transitions.

- $L = \{ \Sigma\Sigma0\Sigma \} \quad \Sigma = \{0, 1\}$
- $L = \{a^n b^n \mid n >= 0 \}$
- $L = \{a^n b^n c^n \mid n >= 0 \}$
- $L = \{a^n b^m c^p \mid n, m, p > 0, \; p = n - m \}$
- $L = \{ w \mid |w| \text{ is even} \}, \Sigma = \{1\}$
- $L = \{ w \mid |w| \text{ a power of } 2 \}, \Sigma = \{1\}$
- $L = \{ w \mid |w| \text{ is prime} \}, \Sigma = \{1\}$

Transducer

- TM produces output (to the tape)

- A function F with domain D is Turing-Computable if there exists a TM, M, such that the configuration q_0w yields q_{accept}, F(w) for all $w \in D$.

  x = number in base 1, F(x) = 2x

  x = 111 2x = 111111

Transducer

- x, y positive integers in base 1
- design TM that computes x+y