CS310

Non-Context-Free Languages

Sections: 2.3 page 123

October 24, 2014

Pumping Lemma

- For regular languages

Pumping Lemma (take two)

Theorem: For any CFG there is an equivalent grammar in CNF.

Pumping lemma (CFG): Suppose A is a CFG. There exists a number *p* such that

if $s \in A$ and $|s| \geq p$

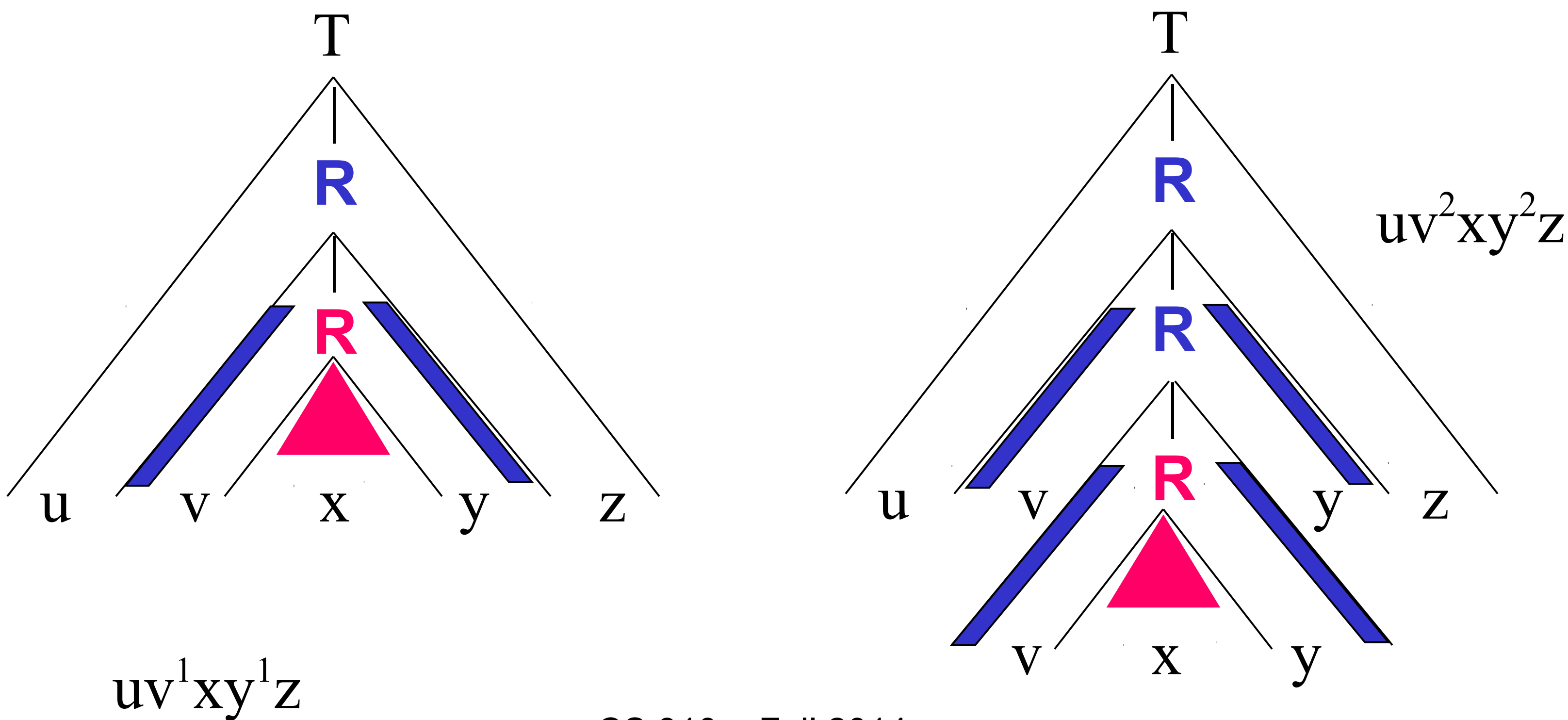
then $s = uvxyz$ where

$uv^ixy^iz \in A, i \geq 0$

$|vy| > 0$

$|vxy| \leq p$

Pumping a Parse Tree



Proof

Suppose A is a CFG in CNF and $s \in A$,

$|s| \geq p = 2^{|V|+1}$

2 ? |V|?

The height of the parse tree for s is ?

Example

$L = \{a^i b^i \mid i \geq 0\}$

a PDA **can** represent this. Why?

Pumping Lemma:

s =

u=

v=

x=

y=

z=

Example

$L = \{a^i b^i c^i \mid i \geq 0\}$

a PDA cannot represent this. Why?

Pumping Lemma:

s =

u=

v=

x=

y=

z=

Example

$L = \{a^i b^j c^k \mid k \geq j \geq i \geq 0\}$

a PDA cannot represent this. Why?

Pumping Lemma:

s =

u=

v=

x=

y=

z=

Example

$L = \{ ww \mid w \in \{0, 1\}^* \}$ Pump-able?

$s =$

Example

$L = \{ w \# x \mid w^R$ is substring of x; $w,x \in \{0, 1\}^* \}$

Pump-able?

s=

Operations

- What operations are closed over context-free languages? (P)
  - Union
  - Intersection
  - Complement
  - Kleene Star
  - Concatenation

Exercise Examples

- p 131
  - 2.30 Show the following are not CFL
    - $\{0^n\#0^{2n}\#0^{3n} \mid n >= 0\}$
    - {w#t | w is a substring of t; w,t in{a,b}*}
  - 2.31
    - Let B be the language of all palindromes over {0,1} containing and equal number of 0s and 1s. Show that B is not Context Free.
  - Show that $\{0^n1^n2^n ; n >= 0\}$ is non context-free

    Show that the complement of $\{0^n1^n2^n ; n >= 0\}$ is context-free.

Sipser 2.24: $E = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$

Show that E is a CFL

CFL?

$L = \{w \mid a^x a^z b^z b^x ; x, z \geq 0\}$

$P = \{ w\#x \mid w^R \text{ is substring of } x; w,x \in \{0, 1\}^* \}$

$N = \{ X + Y = Z, X,Y,Z \in \{0, 1\}^*$ each string forms a valid (binary) mathematical expression. $1+1=10\}$

p 131 2.32

2.38 (CFL not closed under perfect shuffle).