CS310

Context Free Languages and Grammars

Sections:2.1 page 99

October 6, 2014

Context Free Grammar

- Another way to represent a language
  - Can represent more languages than a NFA
- Produces a "Context Free Language"
- Pushdown Automata: machine that recognizes a context free language
- Trivia:
  - First used to describe human languages
  - Now used to parse computer languages (C, C++)

Context Free Grammar

- Example

$A \rightarrow 0A1$

$A \rightarrow B$

$B \rightarrow \#$

Productions or Rules to describe a CFG.

Variables: A, B (may appear on LHS and RHS)

Terminals: 0, 1, # (only appear on the RHS)

Start variable: Variable on LHS of top rule

Language:

Example

- A → → 00#11
  - derivation
  - write u $\rightarrow^*$ v if there is a derivation of the string v from u using the grammar, where u and v are strings of terminals and variables
  - 0A1 $\rightarrow^*$ 00#11
  - Parse Tree

Exercise

$R \rightarrow XRX \mid S$

$S \rightarrow aTb \mid bTa$

$T \rightarrow XTX \mid X \mid \varepsilon$

$X \rightarrow a \mid b$

Variables, terminals of G?

Start variable?

- True or false? $T \Rightarrow^* aba$

Formal Definition

- A context free grammar (CFG) G is a 4-tuple (V, Σ, R, S)
  - V finite set of variables
  - Σ finite set of terminals
  - R set of rules of form:

    variable $\rightarrow$ (string of variables and terminals)
  - $S \in V$, start variable
  - The language of the grammar is:

    $L(G) = \{ w \in \Sigma^* \mid S \xrightarrow{*} w \}$

Example

- $L = \{ w \in \{a, b\}^* \mid aa \text{ is a substring} \}$

Find a grammar that generates this language

  - Can we write this as a regular expression?

Constructing a CFG from a Language, L

- Requires some thought and creativity, just like building a Finite Automaton

- Hints:
  - If possible, break L into pieces L= L1 ∪ L2
    - Create grammar for L1 and L2, $S \rightarrow S_{L1} \mid S_{L2}$
  - If L is regular, use regular expression as guide
  - If L is regular, construct DFA then construct CFG:
    - Make variable R_i for each state q_i in DFA
    - Add rule $R_i \rightarrow \varepsilon$ for all $q_i \in F$, $R_i \rightarrow aR_z$ if $\delta(q_i, a) = q_z$
    - R_0 is start where q_0 is start of DFA

Example

- Grammar G_2 on page 101
- Show derivation for "a boy sees a flower"
  - Notice how this statement is non-creepy?
- Show the parse tree

Write the Grammar

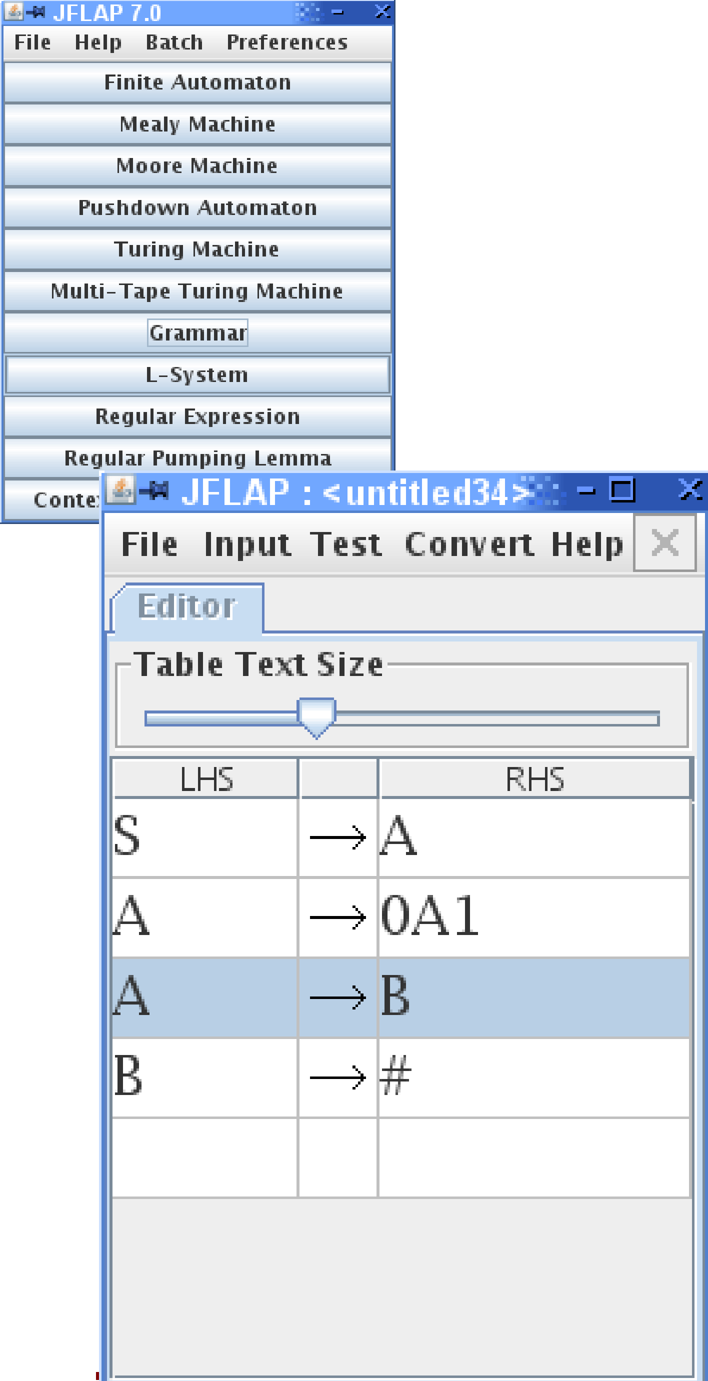
$\Sigma = \{0,1\}$

- {w | w is a binary number greater than 4}
- {w | w is 1^n0^n, n >= 0} ? n >= 1
- {w | w is 1^n0^n, n >= 0, n is even}
- { w | w contains at least three 1s}
- { w | w contains more 1s than 0s}
- {w | |w| is prime}
- $\{a^ib^jc^k \mid i=j \text{ or } i=k\}$ $\Sigma = \{a,b,c\}$
- { w | w is a string of matched ( ) } $\Sigma = \{ (\ ,\ ) \}$

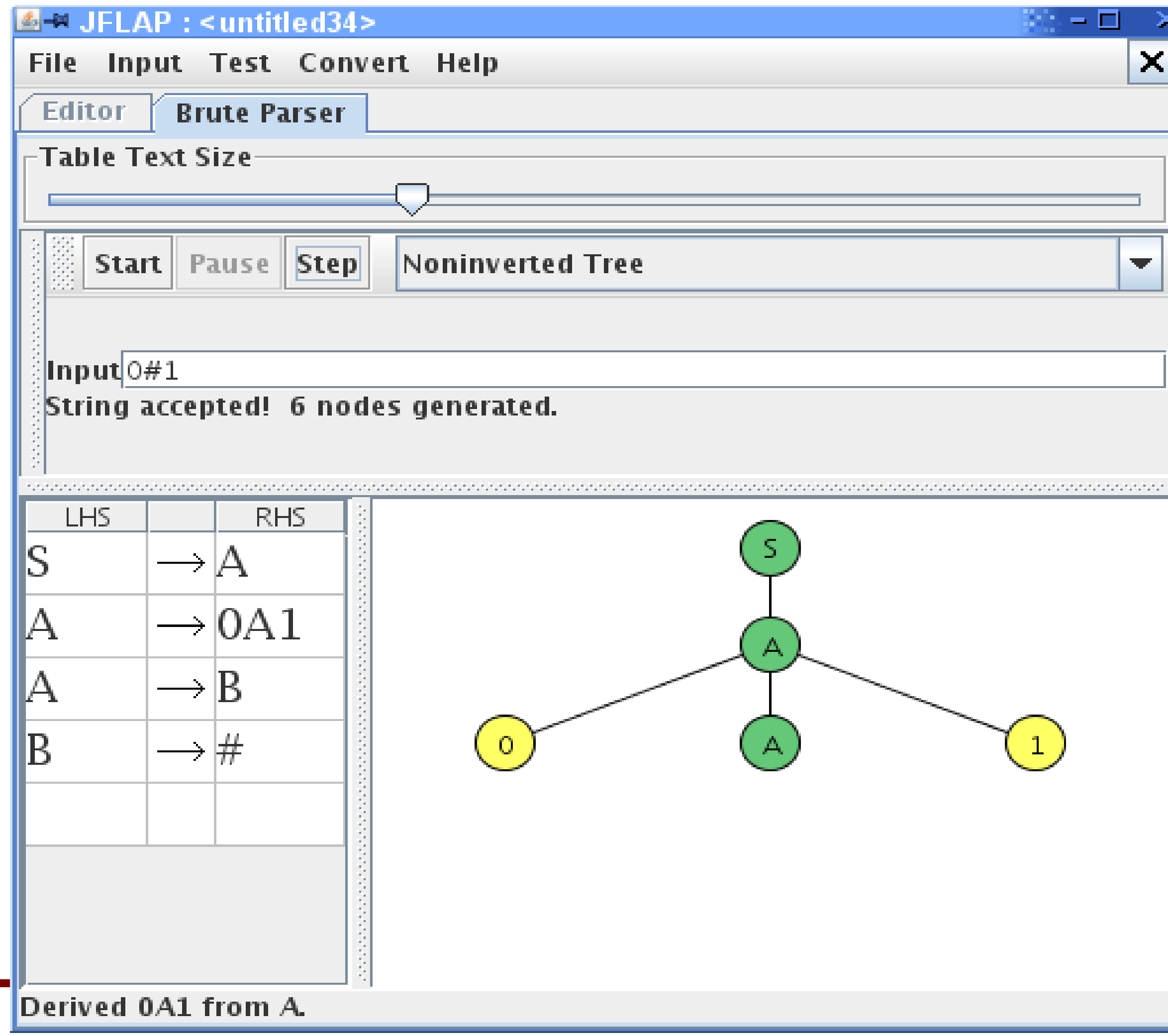
Ambiguous Grammar

- $E \rightarrow E + E \mid E \times E \mid E \mid a$
- Find parse tree for: $a + a \times a$

JFLAP



JFLAP appears to want the start symbol to be S



More examples

$\Sigma = \{0,1\}$

- $\{w \mid |w| \text{ is odd, middle character is } 0\}$
- $\{w \mid w = xyx,\ x \in \Sigma,\ y \in \Sigma^*\}$
- $\{ w \mid w = w^R\}$
- complement of $\{w \mid w = 0^n 1^n ,\ n \geq 1 \}$
- $\{w\#x \mid w^R \text{ is a substring of } x;\ w, x \in \Sigma^*\}$
- $\{w \mid w = 0^{n+m} 1^n,\ n \geq 1,\ m \geq 1\}$
- {w | w contains at least as many *0*s as *1*s}
- $\{w \mid w = 0^{2n} 1^n,\ n \geq 1\}$
- {w | w contains twice as many *0*s as *1*s}