CS310

Pumping Lemma

Sections:1.4 page 77

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String Review

- w is a string
- |w| is
- ww means
- w^n means
- w = xyz, x is a ______________ of w

  z is a ______________ of w

Can
x = ε ?

Non-Regular Languages

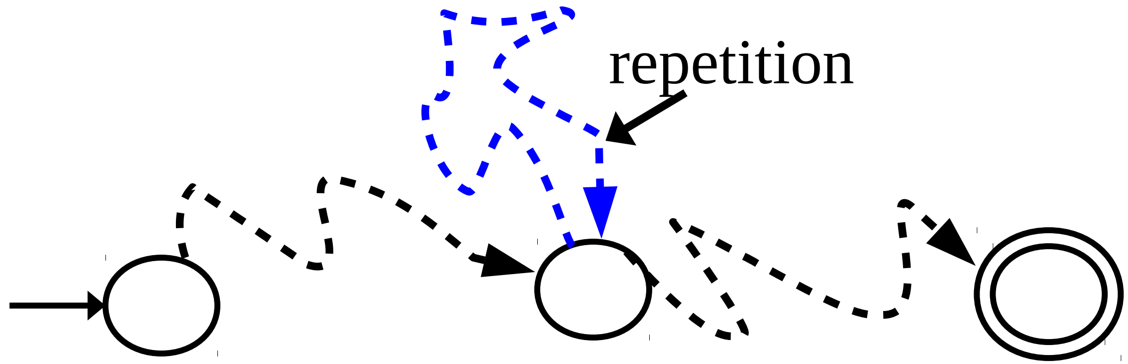
- Languages that *cannot* be represented by a finite automaton
  - Such as?
- How do we prove a language is not regular?
  - What characteristics must a language have to be regular?

C = { w | w has an equal number of 0s and 1s}

D = { w | w has an equal number of occurrences of 01 and 10 as substrings }

Pumping Lemma (Informal)

Pumping: The length of the string could be 'pumped' up by repeating a cycle in the FA, *and the string would still be accepted.*



- All regular languages have a property
  - *the pumping length, p*
- |w| = n, how many states do we go through?

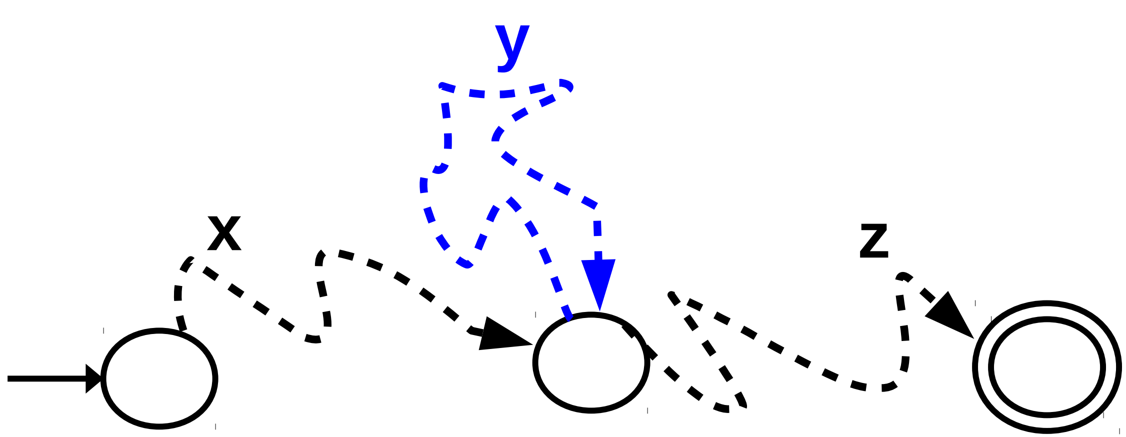
Pumping Lemma (Formally)

- DFA: M= (Q, ∑, δ , q_0, F)

- If |Q| = p and s ∈ L(M) and |s| >= p
  then there exists at least one state that was

  visited twice within the first *p* input symbols

s = xyz



p – pumping length

- For every regular language, some integer *p* exists.

- We do not care what the actual integer value of p is

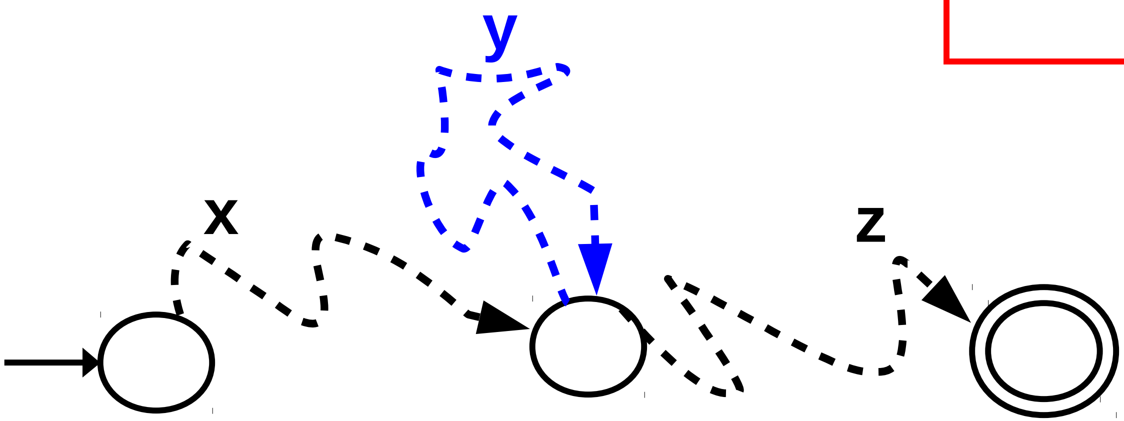
- We will always refer just to *p*

Pumping Lemma (Formally)

- If A is a regular language, then :

s = xyz

- $i >= 0$, $xy^iz \in L(M)$
- $|y| > 0$ (x, z may be ε)
- $|xy| <= p$



Pumping Lemma In Action

- Find a string, $s \in L$, $|s|$ **>=** p, that cannot be pumped to show language L is not regular.
  - Find a string that exhibits the "essence" of nonregularity
  - **Hint:** choose a string that explicitly references the value p!
  - Proof method?
- L = { w | w contains equal number of 0s and 1s }

Practice

- $L = \{ ww \mid w \in \{0, 1\}^* \}$

What string should we chose?

what does ww mean?

Can that be pumped?

Regular vs Non-Regular

$L = \{ 1^* \}$ $\Sigma = \{0,1\}$

$L = \{ 1^*0^* \}$

$L = \{ 1^n \mid n \geq 0 \}$

$L = \{ 0^n1^n \mid n \geq 0 \}$

Examples Galore!

- $L = \{ a^n b^m \mid m > n \}$
- $L = \{ a^n b^m \mid m \text{ is odd}, n \text{ is even}, m>0, n>0\}$
- $L = \{ w1w^R \mid w \in \{0,1\}^*\}$
- $L = \{ a^n b^m \mid m \mathrel{!=} n \}$
- $L = \{ a^{2*n} \mid n > 0\}$
- $L = \{ a^n \mid n \text{ is prime} \}$
- $L = \{ a^n b^m c^{n+m} \mid n, m > 0 \}$
- $L = \{ w^R \mid w \in \{0,1\}^*$, w is a perfect square in binary $\}$
- $L = \{wbbw \mid w \in \{a, b\}^*\}$
- $L = \{ (ac)^n b^m \mid n > m >= 0 \}$
- $L = \{ a^n b^m \mid m >2, n > 2 \}$

Show for each language:

- Are any of these languages regular?

  Can we write any of them as a regular expression?

Practice

- $L = \{ w \mid 1^n0^m1^n, n > 0, m >= 0 \}$ Is L regular?

Which of the following strings are in L and do not violate the pumping lemma?

$s=10^p1$	x =
$s=1^{2p}$	y=
$s=1^p0^p1^p$	z=
$s=0$	xy^iz
$s=1^p01^p$	$i>=0$

More Practice

- $L = \{ w \mid 1^n0^m1^n , 0 < n < 4, m >= 0 \}$ Is L regular?

$L = \{ wy \mid w,y \in \{0, 1\}^*, |w| = |y| \}$

$L = \{ wy \mid w,y \in \{0, 1\}^*, |w| \mathrel{!}= |y| \}$

$L = \{ wy \mid w \in \{a, b\}^*, y \in \{0, 1\}^*, |w| = |y| \}$