

# Theoretical Computer Science CS 310

Chadd Williams

Office Hours:

[chadd@pacificu.edu](mailto:chadd@pacificu.edu)

Mon 2-4pm

202 Strain

Wed 2-4pm

Thur 2-4pm

and by appointment

<http://zeus.cs.pacificu.edu/chadd/cs310f14/>

# Syllabus

<http://zeus.cs.pacificu.edu/chadd/cs310f14/syllabus.html>

- *Introduction to the Theory of Computation* by Michael Sipser, **(Second Edition)**
  - I will assign problems out of this book
  - I expect you to do the readings

## Policies:

- Class starts promptly at 4:45pm
- Assignments are due at the **beginning of class**. Late assignments will not be accepted.
- Programs that do not compile lose 70% (JFLAP)
- The cheating policy is defined in the Pacific Catalog
- Silence all electronic devices
- Participation can raise/lower your grade
  - much board work

# Syllabus

**Grade Distribution**

Homework	20%
Unannounced Quizzes	5%
Exam 1	25%
Exam 2	25%
Final	25%

**Percent Breakdown**

		92-100	A		90-92	A-
88-90	B+	82-88	B		80-82	B-
78-80	C+	72-78	C		70-72	C-
68-70	D+	60-68	D			
0-60	F					

## Tentative Dates:

- Midterm 1, Mon, Sept 29, 2014
- Midterm 2, Fri, November 7, 2014
- Final, Friday Dec 5 (3-5:30pm)

# Today

- Overview of class
- Mathematical Notation
- Proof by Induction

# Who are we?

- Is Computer Science a science?
  - Is it a *natural* science?
  
- What do we study?

# Overview

- What are the fundamental capabilities and limitations of computers?
- How does theory related to programming?
- Complexity Theory
- Computability Theory
- Automata Theory

# Mathematical Notation (Chap. 0)

- Basic notations we will use in this class
  - Page 16 of your book has a partial list (no symbols!)
- Set
- Subset
- Proper Subset

# Sets

- Shorthand for describing a set  
 $\{ n \mid \text{rule about } n \}$



# Set Operations

- What can we do with sets?
- Union
- Intersection
- Complement

# Sets

- Power Set

$\{0, 1\}$

- Cartesian Product (Cross Product)

$\{0, 1\} \times \{a, b\}$

# Sequences/Tuples

- Sequence
- Tuple
  - K-tuple

# Functions

- Object that takes input, produces output

$$f(a) = b$$

- Domain and Range

$$f: D \rightarrow R$$

- Onto

# Functions

- $f : A_1 \times A_2 \times \dots \times A_k \rightarrow R$

$(a_1, a_2, \dots, a_k)$

k-ary

arity

unary (k=1) binary (k=2)

- **Notation**

Infix notation:  $a + b$

Prefix notation:  $\text{add}(a,b)$

# Relations

- Predicate (property)

$$f : D \rightarrow \{\text{TRUE}, \text{FALSE}\}$$

- Relation

$$f : A_1 \times A_2 \times \dots \times A_n \rightarrow \{\text{TRUE}, \text{FALSE}\}$$

- Notation

table

Set

# Equivalence Relations

binary relation

shows that two objects are equal

must satisfy 3 conditions:

1. R is **reflexive** if for every  $x$ ,  $xRx$ ;
2. R is **symmetric** if for every  $x$  and  $y$ ,  
 $xRy$  if and only if  $yRx$ ;
3. R is **transitive** if for every  $x$ ,  $y$ , and  $z$ ,  
 $xRy$  and  $yRz$  implies  $xRz$

# Proof by Contradiction

- Assume the concept is false
- Show this leads to a false consequence
  - tricky to pick exactly the false consequence
- Prove  $\sqrt{2}$  is irrational
  - Assume it is rational:  $\sqrt{2} = m/n$



# Proof by Induction

- Basis

Prove  $P(1)$  is true

- Induction Step

Prove that for each  $i \geq 1$ , if  $P(i)$  is true, then so is  $P(i+1)$ ; *assume  $P(i)$  is true*

- Basis + Induction Step

$P(1)$  is true,  $i = 1$

$P(i+1)$  is true

$P(i+1 + 1)$  is true ...

# Proof by Induction

- Prove:  $1 + 2 + \dots + n = n(n+1) / 2$

for  $n \geq 1$

Basis:

Induction:

# Prove by Induction

- $n^2 \geq 3n, n \geq 3$
  - $F_n$  is the  $n$ th Fibonacci number,  $n \geq 1$ .
    - $F_n = F_{n-1} + F_{n-2}$
    - $F_1 = 1 ; F_2 = 1$
  - $F_{3n}$  is even,  $n \geq 1$
  - $F_{4n}$  is evenly divisible by 3,  $n \geq 1$
- 
-



# Graphs

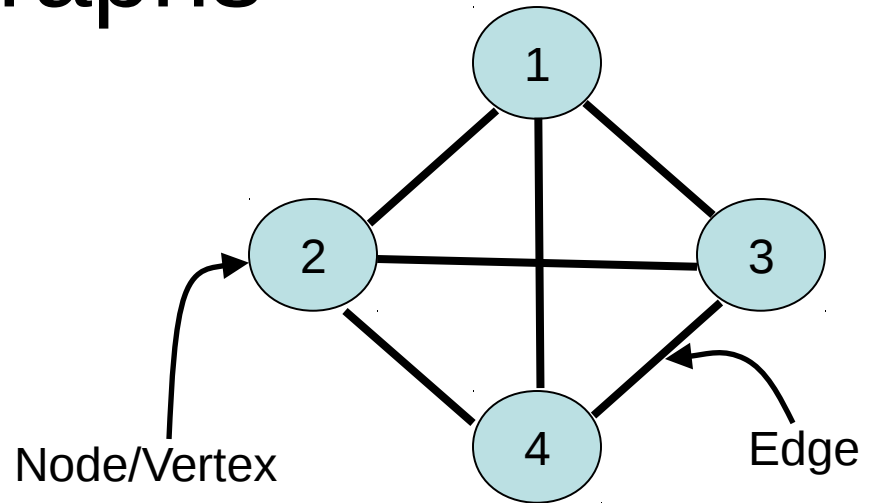
- Graph

Collection of nodes  
and edges

$$G = (V, E)$$

$E =$

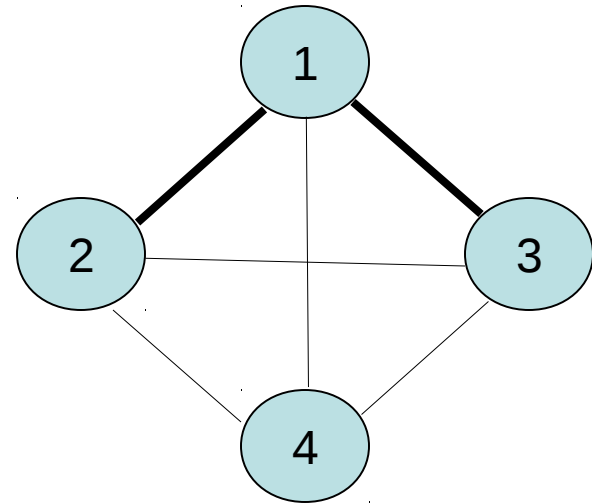
undirected



- Degree of a node

# Graphs

- Subgraph



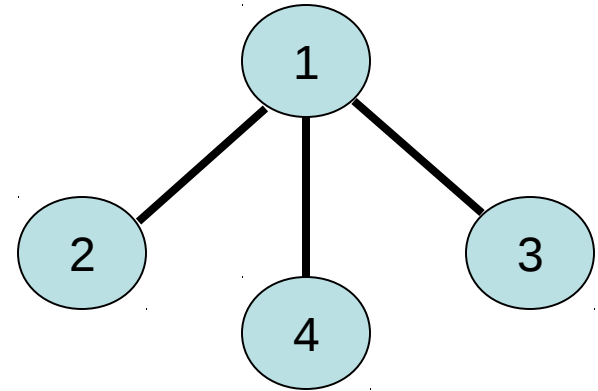
- Path

Connected graph

- Cycle

# Trees

- Connected graph with no simple cycles
- Leaves
- Root



# Directed Graph

- Arrows denote which way an edge goes  
outdegree/indegree

$$G=(V,E)$$

$$V=$$

$$E=$$

Directed path

Strongly connected

