

## Exam 2 Review

**Any topic we discussed in class or that is contained in chapter 0 or 1 or 2 in your book is fair game for the exam.**

List the pieces of the tuple used to define a PDA.

List the pieces of the tuple used to define a CFG.

Prove that the following language is or is not context free:  $\Sigma = \{A,B\}$

$M = \{ w \mid w \text{ contains an even number of As and an odd number of Bs} \}$

$N = \{ w \mid w \text{ contains an unequal number of As and Bs} \}$

$T = \{ w \mid \text{the length of } w \text{ is a power of 3} \} (0, 3, 9, 27, 81\dots) \Sigma = \{A\}$

Describe, using a few English sentences, the main concept that allows the 2<sup>nd</sup> pumping lemma to work (or, why is pumping two variables involved at all?) and how it's different from the 1<sup>st</sup> pumping lemma (why you only pump one variable).

Why is the following string,  $s$ , useless in applying the pumping lemma for the following language  $L = \{c^n a^n b^n \mid n \geq 0\}$   $s = ccaabb$

Show that the following language is or is not context free.  $\Sigma = \{A,B\}$

$L = \{w \mid w \text{ contains more As than Bs} \}$

$M = \{w \mid w \text{ contains an unequal number of As and Bs, and twice as many Cs as Ds} \}$

$Q = L \cap M$  [First, describe in English the language  $Q$ ]

Build a PDA that recognizes the language  $L = \{a^{2^n} b^n \mid n \geq 0\}$

Build a CFG that recognizes the above language. Using your grammar, build the parse tree for **aaaabb**

Build a PDF and CFG that recognizes the following language or show that you cannot.

$L = \{a^{2^n} b^n \mid n \geq 0 \text{ AND } n \% 2 = 1\}$

Build a PDA and CFG for the following language or show that you cannot.

$L = \{a^{2^n} b^n a^q b^{2q} \mid n > 0, q > 0\}$

Imagine that instead of a stack you used a queue in your PDA (a queue pushes on one end and pops from the other end; First In, First Out). Does this change the set of languages you can accept with a PDA?

- 1) Give an example of a language that you could accept with either a stack or a queue in your PDA
- 2) Give an example of a language that you could only accept with a stack in your PDA
- 3) Give an example of a language that you could only accept with a queue in your PDA.
- 4) Could a PDA with a queue accept all regular languages?

Change the following grammar into CNF. First, list all of the nullable variables.

$S \rightarrow ABC \mid \varepsilon$

$A \rightarrow CB \mid ba \mid a \mid \varepsilon$

$B \rightarrow aB \mid bB \mid b$

$C \rightarrow cCC \mid cc$

If A and B are languages, define  $A \bowtie B = \{ xy \mid x \in A \text{ and } y \in B \text{ and } |x| = 2 \cdot |y| \}$ . Show that if A and B are regular languages, then  $A \bowtie B$  is a CFL.

Build a grammar for the following language or show that the language is not a CFL:

$L = \{ w \mid w \text{ is not a palindrome} \}$   $\Sigma = \{a,b\}$

Show that the following language **is or is not** a CFL.

$L = \{ w \mid zw^R ykw^R ; z \in \{0, 1\}^*, k \in \{0, 1\}^*, y \in \{0, 1\}^*, w \in \{0, 1\}^* \}$

The following restriction is part of the CFL pumping lemma:

$|vxy| \leq p$

v and y are the pump-able parts of the string.

The equivalent restriction in the regular language pumping lemma is  $|xy| \leq p$ , with x being the prefix of the string.

1) Why does the regular language pumping lemma include the prefix of the string in the restriction  $|xy| \leq p$ ?

2) Why does the CFL pumping lemma **not** include the prefix (u) of the string in the restriction  $|vxy| \leq p$ ?

$L = \{ w \mid w \text{ is } a^{2^n} b^n; n \geq 1 \}$

Build a grammar for L and prove that your grammar represents the language L correctly. For this proof to go smoothly, you really need to make sure your grammar is as minimal as possible.

Pick any grammar you wrote and turn that grammar into a PDA.

From your book: 2.24, 2.30 2.31, 2.32, 2.33, 2.36, 2.39, 2.41, 2.45

The starred problems are especially hard and likely more difficult than your exam questions.