CS 310 Sample Exam 1: DFA/NFA/Regular Expression/GNFA

Any topic we discussed in class or that is contained in chapter 0 or 1 in your book is fair game for the exam.

Given:
L = { a, b}*
G = {0, 1}*

Give one string in LG, give one string not in LG (neither of these strings should be the empty string if you can avoid it)

Give one string in L ∩ G, give one string not in L ∩ G (neither of these strings should be the empty string if you can avoid it)

Give one string in L ∪ G, give one string not in L ∪ G (neither of these strings should be the empty string if you can avoid it)

Give one string in (LG)*, give one string not in (LG)*, (neither of these strings should be the empty string if you can avoid it)

Use induction to prove the following. Be sure to clearly label your basis and induction step.

Read the definition of Fibonacci numbers from the class notes. Prove that for every n >= 1, f_{4n} is divisible by 3.

Prove

$$1^3+2^3+3^3+...+n^3=\frac{1}{4}n^2(n+1)^2 \quad n\in\mathbb{N}$$

The Alphabet for each language is {0, 1}

1. Build a DFA and give one string in the language and one string not in the language (neither of these strings should be the empty string if you can avoid it)

{ w | w contains substring 1010 }
{ w | w has an even number of 0s and |w| > 4 }
(101)*111(01)*111

2. Build an NFA and give one string in the language and one string not in the language (neither of these strings should be the empty string if you can avoid it)

{ w | every even position of w is 0, |w| >= 2 }
(101)*1(010)*
{ w | |w| > 0 }

For the following three questions, provide the complete machine produced by the lemmas described in your book. **Do no remove non-necessary states and transitions**.

3. Concatenate NFAs
Use the lemma described in the book to concatenate the following two languages:

A = { w | contains substring 1010 }
B = { w | w ends with 00 }

Produce the machine for the language AB

4. Union NFAs
Use the lemma described in the book to union the following two languages:

A = { w | begins with 00 and ends with 11 }
B = { w | w contains the substring 11 }

Produce the machine for the language A U B

5. Kleene Star NFAs
Build a NFA that represents the Kleene star of this language. Show the NFA for the original language (A) and for the Kleene star (A*).

A = { w | 1*0*0101 }

6. Convert a DFA to a Regular Expression using GNFAs

Transform the following DFA to a Regular Expression using the GNFA state removal method. **Show the GNFA after each state removal.**

Sipser Figure 1.6 page 36

7. Produce a Regular Expression
{ w | (starts with 1 and has 1010 as a substring or starts with 0 and has an odd length) and |w| > 0 }

8. Convert an NFA to a DFA

Convert this NFA to a DFA, remove all non-necessary states. Be sure to indicate which set of states in the NFA each state in the DFA represents.

Sipser figure 1.27 page 48

10. Discuss NFA/GNFA/DFA/RES

We have discussed NFAs, GNFAs, DFAs, and regular expressions. Explain, using a few English sentences, the relationships between these concepts.

11. Regular Expressions:

We use three operations to create regular expressions. What are they? Why do we use those three operations? Answer each of these questions using at least one English sentence.

In a spell checker, it is useful to check whether the given word is one symbol away from a word in the dictionary. For a language L, define L' to be the set of all strings obtainable by altering at most one symbol in a string in L. For example, if L is {CAT, DOG} then L' is {AAT, BAT, CAT, …, ZAT, CZT,..., AOG, …, ZOG,....}. Show how to convert an FA for L into an FA for L'.[1]

Use induction to prove that for every undirected graph G, with no loops, the sum of the degrees of all the nodes in G is an even number. Zero is even for this question.

1 Goddard, Wayne, *Introducing the Theory of Computation,* Jones and Bartlett, page 41