CS 310 Final Review Fall 2010

From the book:

p 27: 0.10, 0.11, 0.7, 0.13
p 84: 1.5-1.12, 1.14, 1.16, 1.19, 1.21, 1.31, 1.32, 1.38, 1.41, 1.42, 1.46, 1.47, 1.50, 1.51, 1.53, 1.54, 1.60
p 128: 2.9-2.12, 2.16, 2.17, 2.21, 2.22, 2.24, 2.26, 2.27, 2.28, 2.30, 2.33
p 159: 3.8, 3.9, 3.12-3.16, 3.20, 3.22,
p 183: 4.3, 4.4, 4.6, 4.8, 4.9, 4.11, 4.22, 4.27
p 211: 5.2 5.10
p 294: 7.6 7.7 7.8

Proofs

Prove the following when n in an integer greater than 0:

$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1/4) * n^2 * (n + 1)^2$

Prove the following by using proof by contradiction.

The product of odd integers is odd.

DFA/NFA/Regular Expressions/Pumping Lemma

Explain each element in the 5-tuple used to define a DFA. Which piece of the 5-tuple is different, and how, between a DFA and an NFA.

Build a DFA, an NFA, and a Regular Expression for the following languages or prove that you cannot:

$\Sigma = \{ 0, 1 \}$

{ w | |w| is even }
{ w | w contains an odd number of 0s and even number of 1s; w contains at least one 1 and at least one 0 }
{ w | w contains exactly as many 1s as 0s }
$\{ 0^k 11 \mid k > 2 \}$
$\{ 0^k 1 1^k \mid k > 0 \}$

Build an NFA for the following language and convert it to a DFA. Be sure to list which states in the NFA are represented by each state in the DFA.

$\Sigma = \{ 0, 1 \}$

{ w | w contains the substring 0101 }

Build a DFA for the following language and convert it to a Regular Expression using a GNFA.

{ w | w contains an odd number of ones followed by an even number of zeros followed by any number of zeros and ones }

The class of regular languages is closed under what operations? Prove that fact for one of the operations.

Build DFAs for the following languages and use one of the operations above to combine them.

{ w | w contains exactly 3 1s and any number of 0s}

{ w | w contains an even number of 1s and any number of 0s }

Context Free Languages

Give two strings that can be derived from the following CFG:

```
S -> 0S | S1 | A
A -> 10
```

Build a CFG that generates the following languages or prove that you cannot:

$\Sigma = \{ 0, 1 \}$

{ w | w contains the substring 0101 }

{ w | contains more 0s than 1s }

$\{ 1^k 0^k 1^j \mid k > 0, j > 0 \}$

$\{ ww^R \mid w \in \Sigma^* \}$

$\{ ww^R \mid w = 10^n 1^n, n > 0 \}$

Consider the CFG:

```
S -> ABCD | AC
A -> aA | aB | ε
B -> Bb | Ba | b
C -> cc | CC | ε
D -> d | dDD | ε
```

Is it ambiguous? If so, can you make it unambiguous?

Convert the above (unaltered) CFG to CNF.

Convert the above (unaltered) CFG to a PDA.

Build the parse tree (or show that you can't) for the following string using the (unaltered) grammar above:

bbddddd

Parse the above string using the PDA you built. Show the contents of the stack at each step.

Build FIRST and FOLLOW for the above CFG. Build the LL(1) parse table.

Give

an example of a language that is context free but not regular

an example of a language that is regular but not context free

or explain why you cannot.

Turing Machines

Build a complete, deterministic Turing Machines (full state diagram with all transitions) for the following languages or show that you cannot. You may use any number of tapes:
Be sure to write an English language description of your algorithm before you begin.

$\{ ww^R \mid w = 10^n1^n, n > 0 \}$
$\{ w \mid w \text{ contains more 1s than 0s} \}$
$\{ 1^k0^k1^{(2k)} \mid k > 0 \}$

Explain, using English sentences, what the Halting problem is and why it is important in Computer Science.

Explain, using English sentences, what a correspondence is and how it is used to compare the sizes of infinite sets.

State the acceptance problem for DFAs (A_{DFA}). How is this related to $A_{\text{Regular Expression}}$?

Time Complexity

If TM, M, has a precise runtime of: $5n^3 + 99n^4 + 2n^2 + 99$, what is its O() runtime?
If TM, N, has a precise runtime of: $2n^5+6$, what is its O() runtime?
Which TM, N or M, has a higher time complexity?

For a function $f(n) = n^3 + n^2$, $f(n) = O(n^3)$. When $n = 2$, $f(n) = 12$ and $O(n^3) = 8$. Explain how this can be if $O(n^3)$ is supposed to be an upper bound on f(n), or show that $f(n) \neq O(n^3)$.

What is the Big Oh runtime of your Twos-Complement TM in homework 5?

What is the Big Oh runtime of your Adder TM in homework 5?

Explain what it means for a language to be in class P.

Explain what it means for a language to be in class NP.

Why is the question: Does P = NP? important to Computer Science?