CS310

Finite Automata
Sections: 1.1, 1.2 page 44

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Quick Review

• Deterministic Finite Automata:
  5-tuple \((Q, \Sigma, \delta, q_0, F)\)
  \(Q\): finite set of states
  \(\Sigma\): alphabet (finite set)
  \(\delta\): transition function \((\delta: Q \times \Sigma \to Q)\)
  \(q_0\): start state
  \(F\): accepting states (subset of \(Q\))

• Language \(A\) is \textit{regular} if there exists a Finite Automata that recognizes \(A\).
Regular Language

• Determinism?

• Regular language
  – Example?

  – Example of non-regular language?
Regular Operations on Languages

• Given two languages, A,B, we can create new languages in a variety of ways:
  – What operations have we seen?
Examples

$\Sigma = \{0, 1\}$  
$A = \{w \mid w \text{ ends in } 1\}$  
$B = \{w \mid w \text{ begins with } 00\}$

$A \cup B =$

$AB =$

$A^* =$

$A \cap B =$

$\overline{A} =$
Closure of Regular Languages

• A set is *closed* under some operation
  – Examples?

• *Regular operations*
Proof

• Theorem 1.25: The class of regular languages is closed under the union operation.

If $A$ and $B$ are regular languages, so is $A \cup B$

What do we need to prove?

What does regular mean?

What does it mean for $A \cup B$ to be regular?
Build the machine

\[ \Sigma = \{0, 1\} \]
\[ A = \{ w \mid w \text{ contains a 1 in the penultimate position} \} \]

\[ A = \{ \} \]
Nondeterminism

• Nondeterministic Finite Automata:
NFA

• $\varepsilon$ transitions

• Why would we ever use this?
Example

• Does this NFA accept 010110?
• What sequence of states does it go through?
Proof

• Theorem 1.26: The class of regular languages is closed under the concatenation operation.

If A and B are regular languages, so is AB.

What do we need to prove?
What does regular mean?
What does it mean for AB to be regular?
Problems?
Examples

A = \{north, south\} \quad B = \{east, west\}

w = northeast is in AB

many ways to break down this string

If the AB machine breaks the string as nor
t and heast the string will not be accepted

A = \{w \mid w = \text{begins with 1 ends with 0}\}
B = \{w \mid w = \text{begins with 0 ends with 1}\}

w = 1000011
Proof

• Theorem 1.26: The class of regular languages is closed under the concatenation operation.
  
  If A and B are regular languages, so is AB. NFA!

  We will get back to this after more practice with NFAs.