CS310

Variants of Turing Machines

Section 3.2

November 12, 2008
Formal Definition (1 tape)

- **7-tuple**
- \( \{ Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}} \} \)
- **Q**: set of states
- **\( \Sigma \)**: input alphabet, not containing the blank character: \( \forall \)
- **\( \Gamma \)**: tape alphabet, \( \forall \in \Gamma \) and \( \Sigma \subseteq \Gamma \)
- **\( \delta \)**: \( Q \times \Gamma \rightarrow Q \times \Gamma \times \{ L, R \} \) is the transition function
- **\( q_0 \in Q \)**: start state
- **\( q_{\text{accept}} \in Q \)**: accept state
- **\( q_{\text{reject}} \in Q \)**: reject state, \( q_{\text{accept}} \neq q_{\text{reject}} \)
Multiple Tape Turing Machine

- For k tapes
  - input string is on tape 1
- Change

\[ \delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\} \]

To

\[ \delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k \]
Example

• Construct a two-tape Turing Machine to accept $L=${$a^n b^n$ | $n \geq 1$}

• Conceptually what do we want to do?
Theorem

• Every multi-tape Turing Machine has an equivalent single tape machine
  – adding extra tapes does not add power to the Turing Machine

• Proof Idea: Simulate multi-tape TM as single tape TM
Nondeterministic TM

• Just like NFA, can take multiple transitions out of a state
  – often easier to design/understand

• Design a TM to accept strings containing a c that is either preceded or followed by \( ab \)

• We can think of this computation as a tree
  – each branch from a node (state) represents one nondeterministic decision (for a single input character)
Theorem

- Every nondeterministic TM, N, has an equivalent deterministic TM, D

Proof Idea:
- use a 3 tape TM (we can convert this to a one tape TM later)
  - tape 1: input tape (read-only)
  - tape 2: simulation input/output tape of the current branch of the n-d TM
  - tape 3: address tape (based on the tree) to keep track of where we are in the computation
Practice

\{ a^ib^jc^k \mid i > j > 0; \ k = 2i \}
\{ w w^R \mid \text{ww}^R \text{ is odd}, \ w \in \{0,1\}^* \}
the complement of \{w w^R \mid w \in \{0,1\}^* \}

multiplication of two numbers in base 1:
11111 * 11 produces 11111111111