CS310

Pumping Lemma
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Non-Regular Languages

• Languages that *cannot* be represented by a finite automaton
  – Such as?

• How do we prove a language is not regular?

\[ \mathcal{C} = \{ w | w \text{ has an equal number of 0s and 1s} \} \]
\[ \mathcal{D} = \{ w | w \text{ has an equal number of occurrences of 01 and 10 as substrings} \} \]
Pumping Lemma (Informal)

Pumping: The length of the string could be ‘pumped’ up by repeating the cycle, and the string would still be accepted.

- All regular languages have a property
  - the pumping length, $p$
- $|w| = n$, how many states do we go through?
Pumping Lemma (Formally)

- DFA: $M = (Q, \Sigma, \delta, q_0, F)$

If $|Q| = p$ and $s \in L(M)$ and $|s| \geq p$ then there exists at least one state that was visited twice within the first $p$ input symbols

$s = xyz$
Pumping Lemma (Formally)

- If $A$ is a regular language, then:

$s = xyz$

- $i \geq 0$, $xy^iz \in L(M)$
- $|y| > 0$ (x, z may be $\varepsilon$)
- $|xy| \leq p$
Pumping Lemma In Action

• Find a string, $s \in L$, $|s| \geq p$, that cannot be pumped to show language $L$ is not regular.
  – Find a string that exhibits the “essence” of nonregularity
  – Proof method?

• $L = \{ w \mid w \text{ contains equal number of 0s and 1s} \}$
Pumping Lemma in Action

- \( L = \{ w \mid w \text{ contains equal number of 0s and 1s} \} \)

use a different string:
Can that be pumped?

\[
\begin{align*}
s &= \\
x &= \\
y &= \\
z &= 
\end{align*}
\]
Practice

- \( L = \{ \text{ww} \mid w \in \{0, 1\}^* \} \)

What string should we choose?

what does \( \text{ww} \) mean?

Can that be pumped?