CS310

Nondeterministic Finite Automata
Sections: 1.2 page 47

September 11, 2006
Nondeterminism

• What?
  State diagram can have none, one, or many exiting arrows for each symbol at each state
  ε-transition automatically taken without any input

• Why?
  Recognizes same languages as DFA
    (can convert NFA to DFA)
  Often easier to build
  Need machine that will run processes simultaneously
Example (1.30)

- Accept string of at least length three that contains a 1 in the third from end

\[ \Sigma = \{0, 1\}, \quad \Sigma^*1(0U1)(0U1) \]

What makes this difficult for a DFA?

Equivalent DFA takes 8 states. Why 8?
Example

Accept: \( \varepsilon, a, baba, baa \)
Reject: \( b, bb, babba \)
Formal Definition of NFA

- 5 tuple ( Q, Σ, δ, q₀, F)

  \[ \Sigma_\varepsilon = \Sigma \cup \{\varepsilon\} \]

  \[ \delta : Q \times \Sigma_\varepsilon \rightarrow P(Q) \quad (DFA: \delta : Q \times \Sigma \rightarrow Q) \]

  \( P(Q) \) is the power set of Q. The collection of all subsets of Q. \( P(Q) \) has \( 2^{\mid Q \mid} \) members

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>\varepsilon</th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>{q1,q2}</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>q2</td>
<td>ø</td>
<td>{q3}</td>
<td>{q3}</td>
</tr>
<tr>
<td>q3</td>
<td>ø</td>
<td>{q4}</td>
<td>ø</td>
</tr>
<tr>
<td>q4</td>
<td>{q4}</td>
<td>{q4}</td>
<td>ø</td>
</tr>
</tbody>
</table>
Formal Definition of Computing for NFA

• Given a machine $M = (Q, \Sigma, \delta, q_0, F)$ and a string $w = w_1 w_2 \ldots w_n$ over $\Sigma$, then $M$ accepts $w$ if there exists a sequence of states $r_0, r_1 \ldots r_n$ in $Q$ such that:
  
  – $r_0 = q_0$ : $r_0$ is the start state
  – $\delta (r_i, w_{i+1}) = r_{i+1}$, $i=0, \ldots, n-1$ : legal transitions
  – $r_n \in F$ : stop in an accept state

• Identical to the definition for the DFA!
Practice

• Construct a NFA with three states that recognizes \( \{ w \mid w \text{ ends with two 0s} \} \)
\[
\Sigma = \{0, 1\}
\]
Practice

• Construct a NFA with six states
  \( \{w \mid w \text{ even \# 0s OR exactly two 1s}\} \)
  \( \Sigma = \{0, 1\} \)
Practice

• Construct a NFA with three states

$0^*1^*0^*0$

$\Sigma = \{0, 1\}$